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Investigating the Theoretical Structure of the Differential Ability Scales—Second Edition Through Hierarchical Exploratory Factor Analysis

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Abstract

When the Differential Ability Scales-Second Edition (DAS-II; Elliott, 2007a) was developed, the instrument's content, structure, and theoretical orientation were amended. Despite these changes, the Technical Handbook (Elliott, 2007b) did not report results from exploratory factor analytic investigations, and confirmatory factor analyses were implemented using selected subtests across the normative age groups from the total battery. To address these omissions, the present study investigated the theoretical structure of the DAS-II using principal axis factoring followed by the Schmid-Leiman (1957) procedure with participants from the 5 to 8 year-old age range to determine the degree to which the DAS-II theoretical structure proposed in the Technical Handbook could be replicated. Unlike other age ranges investigated where at most 14 subtests were administered, the entire DAS-II battery was normed on participants ages 5 to 8, making it well-suited to test the full instrument's alignment with theory. Results suggested a sixfactor solution that was essentially consistent with the CHC-based theoretical structure suggested by the test publisher and simple structure was attained. The only exception involved two subtests (Picture Similarities and Early Number Concepts) that did not saliently load on a group factor. Implications for clinical practice are discussed.

Keywords: DAS-II; General Intelligence; Cattell-Horn-Carroll Theory; Schmid-Leiman Orthogonalization; Exploratory Factor Analysis; Higher Order Factor Analysis

Investigating the Theoretical Structure of the Differential Ability Scales—Second Edition Through Hierarchical Exploratory Factor Analysis

The Differential Ability Scales—Second Edition (DAS-II; Elliott, 2007a) is a test of cognitive ability that is individually administered to children and adolescents ages 2 through 17 years. The DAS-II is a revision of the original DAS (Elliott, 1990) which itself was predicated upon the British Abilities Scales (BAS; Elliott, Murray, & Pearson, 1979). The *Introductory and Technical Handbook* (Elliott, 2007b; hereafter "*Technical Handbook*") indicated that the development of the DAS-II was guided by the Cattell-Horn-Carroll theory of cognitive abilities (CHC; Carroll, 1993; Cattell & Horn, 1978; Horn & Cattell, 1966; Schneider & McGrew, 2012). The *Technical Handbook* also states that the DAS-II has the capacity to measure 7 CHC factors (i.e., Crystalized Intelligence/Knowledge [Gc], Fluid Intelligence/Reasoning [Gf], Visual-Processing [Gv], Short-Term Memory [Gwm], Long-Term Retrieval [Glr], Processing Speed [Gs], and Auditory Processing [Ga]), although the Ga factor contains a single subtest (e.g., Phonological Processing).

The *Technical Handbook* reports that the DAS theoretical orientation, structure, and subtest content were revised between the first and second editions. Previously, the DAS was guided by "...an eclectic number of theoretical perspectives" (Elliott, 2012, p. 338), but now is explicitly guided by CHC theory (Elliott, 2012). Structurally, the DAS-II removed the previous edition's achievement related clusters and added a working memory cluster. The subtest content also changed from the prior edition. Three subtests were deleted (i.e., Word Reading, Spelling and Basic Number Skills), two subtests were combined into a single measure (i.e., Pattern Construction and Block Building), and several new subtests were added (i.e., Rapid Naming, Recall of Digits Backward, Recall of Sequential Order, and Phonological Processing).

When a cognitive ability instrument such as the DAS-II is developed or significantly revised, research investigating internal structure is a necessary step in assessing construct validity (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 2014; Dombrowski, 2015a). This is most often accomplished by conducting factor analysis (i.e., exploratory factor analysis [EFA] and/or confirmatory factor analysis [CFA]). Although it has been suggested that EFA is preferred as a first step prior to using CFA when there are significant refinements to a measurement instrument and the structuring of variables is less clear (e.g., Carroll, 1998; Haig, 2005; Gerbing & Hamilton, 1996; Schmitt, 2011), the test publisher relied exclusively on CFA to furnish evidence of internal structure¹.

The *Technical Handbook* posited that CFA procedures were justified because the DAS-II was based upon the previously established instrument; however, comparability with a previous version and the singular reliance upon CFA procedures to determine the latent structure of a newly developed or revised instrument has been criticized in the literature (Beaujean, 2015a; Canivez, 2013; Dombrowski & Watkins, 2013; Frazier & Youngstrom, 2007; Strauss, Spreen, & Hunter, 2000). EFA and CFA are considered complementary procedures, but they provide answers to different empirical questions (Gorsuch, 1983; Carroll, 1998). When these two procedures produce results that are consistent, then greater confidence can be engendered in the structure of an instrument. Unfortunately, EFA analyses on the DAS-II were not reported despite the significant change to the instrument's content and theoretical structure. Instead, within the *Technical Handbook*, separate CFAs were conducted for various age groups (e.g., ages 3–4, 5–11, and 6–17:11) to determine the DAS-II factor structure. Across these age ranges,

¹ Within the *Technical Handbook*, there is language to indicate that "exploratory" analyses were conducted; however, the results of these analyses are not reported.

14 or fewer core and diagnostic subtests were administered to participants and subsequently investigated. Interestingly, the *Technical Handbook* reports that the entire DAS-II battery was normed on participants ages 5 to 8, making it well suited to test the full instrument's alignment with theory. However, the test publisher did not report separate analyses with participants in the 5 to 8 year-old age range.

There are additional DAS-II factor analytic results that would be useful additions to the *Technical Handbook* (and the corpus of the DAS-II structural validity literature). These include model-based reliability estimates (i.e., omega hierarchical and omega hierarchical subscale), percentages of variance captured by higher- and lower-order factors, communality estimates, and first-order factor correlations from EFA. The methodological research base in exploratory factor analysis (e.g., Carroll, 1993, 1995, 1997, 2003; Gorsuch, 1983; McClain, 1996; Ree, Carretta, & Green, 2003; Thompson, 2004) has suggested the inclusion of this information when presenting factor analytic results as it aides users in determining how an instrument should be interpreted and how much interpretive emphasis should be placed upon higher- and lower-order factors. These statistics were not provided in the DAS-II *Technical Handbook*, suggesting that understanding of the DAS-II and its relationship with CHC theory is presently incomplete.

A survey of the DAS-II structural validity literature beyond the *Technical Handbook* reveals one study that used the 5 to 8 year old age range in an analysis (using normative data from the test publisher). The goal of that study was to investigate the invariance of the DAS-II structure across the three age ranges reported in the *Technical Handbook*. Given the differences in test content and structure across the age span of the test, Keith et al. (2010) used the participants in the 5 to 8 year age range as an anchor for investigating the invariance of the DAS-II across the three other age ranges presented in the *Technical Handbook*. Keith et al. found

evidence for invariance of a six-factor (Gc, Gf, Gv, Gwm, Glr, Gs) structure². However, the final validation model contained additional complexity that was not modeled by the test publisher (i.e., correlated factors and subtests within a higher-order structure). Additionally, to obtain the measurement model, the authors tested over 20 rival models with different post-hoc combinations of correlated errors and cross-loadings based on the results from modification indices. This may well suggest complexity in the measurement model that would have benefitted from an *a priori* EFA in order to suggest such CFA structural features and avoid the concern of hypothesizing after results are known (Cucina & Byle, 2017; Horn, 1989).

Two other studies that investigated the structure of the core DAS-II battery. Canivez and McGill (2016) investigated the core DAS-II battery and its alignment with the three DAS-II verbal, nonverbal, and spatial ability clusters across the three age ranges (3–4, 5–6, 6–17) reported in the *Technical Handbook*. Using EFA and the Schmid and Leiman (SL; 1957) procedure, Canivez and McGill found evidence that the DAS-II core battery reflected a three factor instrument (although the results of several extraction criteria suggested one factor) but they noted that the general factor absorbed a significant portion of the variance and the instrument was best interpreted as a measure of general ability across the age ranges studied because little unique variance was apportioned to lower-order group factors. Dombrowski, Golay, McGill and Canivez (2017) obtained normative data from the test publisher and investigated the six DAS-II core battery using Bayesian structural equation modeling (BSEM) within the 5 to 8 age range. Dombrowski et al.'s results revealed plausibility of a three-factor model, consistent with publisher theory, expressed as either a higher-order (HO) or a bifactor (BF) model. The BSEM findings also yielded an alternative structure with the best model fit, a

² It should be noted that some of the latent factors identified in the *Technical Handbook's* CFAs as well as the invariance investigation furnished by Keith et al. (2010) are not consistent with the cluster scores on the DAS-II.

two-factor BF model with Matrices (MA) and Sequential & Quantitative Reasoning (SQ) loading on *g* only with no respective group factor loading.

Purpose of the Current Study

The present study extends previous research by assessing the factor structure with exploratory factor analyses of the full DAS-II test battery for participants in the 5 to 8 age range. Although the DAS-II was reformulated based upon a new theory (e.g., CHC theory), and the structure and subtest content was revised, EFA results for the full battery were not reported in the *Technical Handbook* even though it serves as an anchoring point for the entire battery. The present study fills this gap in the literature by investigating the theoretical/factor structure of the full DAS-II battery across the only age range where the entire battery was administered to participants. Given the widespread use of the DAS-II in clinical practice, the results of this study portend to assist with the understanding of the theoretical structure and resulting evidence-based assessment practices that have been suggested for the DAS-II.

Method

Participants

Normative data for the DAS-II were collected from a nationally representative sample of 3,480 participants from ages 2 through 17. The DAS-II *Technical Handbook* reports detailed demographic characteristics including controlled matching to the 2002 U.S. Census on such variables as sex, ethnicity/race, and parental education level. Standardization sample raw data with individual participant performance from the 5 to 8 age range (N = 787) were provided by NCS Pearson, Inc. to conduct independent analyses.

Measurement Instrument

The DAS-II is an individually administered test of cognitive ability for children and adolescents ages 2 through 17 years. The *Technical Handbook* and subsequent writings (e.g., Elliott, 2012) indicate that the development of the DAS-II was guided in large measure by the CHC model of cognitive abilities. As previously discussed, the DAS-II is a complex instrument containing combinations of supplemental and diagnostic subtests throughout the age range that yield six additional first—order cluster scores. These measures, however, are not used to calculate the higher-order GCA composite or lower-order cognitive clusters. Please see the *Technical Handbook* or Elliott (2012) for a discussion of these clusters as well as a description of subtest demands.

Procedure

The 20 DAS-II subtests available for the 5 to 8 year old participants were examined using several EFA methodological approaches. Initially, Bartlett's Test of Sphericity (Bartlett, 1954) and the Kaiser-Meyer-Olkin (KMO; Kaiser, 1974) statistic were used to examine the intercorrelation matrix and ensure that it was appropriate for factor analysis. Second, the intercorrelation matrix was subjected to principal axis factoring (PAF; Cudeck, 2000; Fabrigar, Wegener, MacCallum, & Strahan, 1999; Tabachnick & Fidel, 2007) with promax rotation (k = 4; Tataryn, Wood, & Gorsuch, 1999) because it was assumed that the extracted factors would be correlated (Gorsuch, 1983; Schmitt, 2011; Tabachnik & Fidell, 2007). Pattern coefficients of .30 or higher were considered salient (Child, 2006; Schmitt, 2011). Third, several empirical factor extraction criteria were examined (Gorsuch, 1983) along with factor interpretability and whether the results complied with desired simple structure (Thurstone, 1947). Accordingly, the visual scree test (Cattell, 1966), Horn's parallel analysis (HPA; Horn, 1965), the Bayesian Information Criterion (BIC), and the minimum average partial test (MAP; Velicer, 1976) were examined.

MAP was conducted using O'Conner's (2000) SPSS syntax. HPA and BIC were calculated using the R Statistical Programming Language (R Development Core Team, 2017), using the psych (Revelle, 2012) package for factor extraction. Additionally, six factors were extracted to see if the publisher proposed model could be replicated. Finally, a second-order factor analysis followed by the Schmid-Leiman (SL; 1957) procedure applied to the oblique first-order factors assisted in elucidating the structure of the DAS-II. Wolff and Preising's (2005) SPSS code was used for the SL procedure.

Coefficients omega-hierarchical (ω_H) and omega-hierarchical subscale (ω_{HS}) (Reise, 2012) were estimated using the *Omega* program (Watkins, 2013). Omega-hierarchical reflects the model-based reliability estimate for the general intelligence factor with variability of group factors removed. The ω_{HS} coefficient estimates model-based reliability inherent in a group factor with all other group *and* general factors removed (Brunner, Nagy, & Wilhelm, 2012; Reise, 2012). Omega estimates (ω_H and ω_{HS}) may be obtained from decomposed variance estimates from bifactor or approximate bifactor models (when factors are not complexly determined) including the SL. Reise (2012) and Reise, Bonifay and Haviland (2013) note that omega coefficients should exceed .50, but .75 is preferable to indicate sufficient construct-based reliability for independent interpretation of a group or hierarchical factor.

Results

Both Bartlett's Test of Sphericity [Bartlett, 1950; χ^2 (190) = 7,246.44, p < .0001] and the Kaiser-Meyer-Olkin (Kaiser, 1974) statistic (.95) indicated that the correlation matrix was appropriate for factor analysis. Measures of sampling adequacy for each variable also appeared to be within reasonable limits suggesting that the analytical techniques used in this study were appropriate.

Factor Extraction Criteria Comparison

Horn's (1965) parallel analysis indicated the extraction of either five factors (common factors) or three factors (components). Adjusted BIC suggested extraction of five factors. The MAP (Velicer, 1976) criterion recommended extraction of two factors. The visual scree test provided evidence for 5 or 6 factors. On the bases of these results, three and five factors were extracted and examined. A six factor solution was also extracted in accord with the theoretical structure presented in the *Technical Handbook*. After examination of local fit, the six factor extraction was deemed most interpretable as it had the greatest alignment with CHC theory and the most plausible results.

Exploratory and Hierarchical Factor Analysis

Principal Axis Factoring. Table 1 presents the PAF analyses for the correlation matrix according to a six-factor extraction. Tables A1 and A2 (online supplement) present the respective results of the PAF analysis with three- and five-factor extractions. These tables also include the correlations among the extracted factors, communality estimates, uniqueness, pattern and structure coefficients, eigenvalues for retained factors, and percentage of variance accounted for by each factor. Table 1 and Tables A1 and A2 indicate that the first factor accounted for 42.4% of the variance while the second factor accounted for 6.5% of the variance. For the six-factor extraction correlations among the extracted factors ranged from .41 to .74 (*Median* = .49). The six-factor model appeared to produce the best solution with all subtests obtaining salient factor pattern coefficients on their theoretically consistent factor except for Picture Similarities which had no salient pattern coefficients on any factor. Also, the six-factor model produced a generally simple structure with only one subtest cross-loading. Pattern Construction-Alternative obtained a salient pattern coefficient on its theoretically consistent factor but had a small cross-loading on

the fluid reasoning factor. For the five-factor extraction the correlations among the five factors ranged from .39 to .75 (*Median* = .67). For the three-factor extraction the correlations were .42, .44, and .75. Moderate to high correlations among factors, along with extant intelligence test theory, suggests the likely presence of a higher-order factor, which would benefit from extraction and examination (Gorsuch, 1983; Thompson, 2004). This was accomplished through the application of the SL orthogonalization procedure.

Schmid-Leiman Hierarchical Analyses. The SL results for the DAS-II with six-factors are presented in Table 2. The general factor contained 36.5% of the total variance and 69.2% of the common variance, surpassing the variance accounted for by the lower-order group factors (2.1% to 3.2% total variance; 3.9% to 6.0% common variance). The general factor also contained between 18.7% and 47.1% (*Median* = 37.9%) of individual subtest variance. The first-and second-order factors combined to measure 52.7% of DAS-II variance, reflecting 47.3% unique variance. This result provided evidence for a strong manifestation of a general intelligence factor in the DAS-II where the combined influence of general intelligence and uniqueness surpassed the contributions made by the first-order group factors.

For the three- and five-factor SL solutions (Online Supplement Tables A3 and A4), the general factor accounted for 34.6% and 36.3% of the total variance, respectively, and 71.9% and 71.5% of the respective common variance, exceeding that accounted for by the group factors: (4.2% to 4.9% total variance [3-factor]; 2.2% to 3.6% total variance [5-factor]; 8.8% to 10.0% common variance [3-factor]; 4.2% to 7.0% common variance [5-factor]). The general factor also accounted for between 16.2% and 48.7% (<math>Mdn = 34.6%) of individual subtest variance in the three-factor analysis, and 17.8% and 57.5% (Mdn = 38.7%) of individual variance in the five-factor analysis. The general and group factors combined to measure 48.2% and 50.8% of the

respective DAS-II variance, reflecting 51.8% and 49.2% respective unique variance in the three-and five-factor solutions. Across all analyses, and despite the extraction of three-, five-, or six-factors, the results demonstrated a potent general intelligence factor in the DAS-II where the joint contribution of general intelligence and uniqueness exceeded the contribution made by the group factors.

Omega Estimates. The SL results presented in Table 2 were used to estimate omegahierarchical (ω_H) and omega-hierarchical subscale (ω_{HS}) coefficients. The ω_H coefficient for general intelligence (.88) was high and appropriate for confident scale interpretation of a unitweighted composite score. The ω_{HS} coefficients for the six DAS-II group factors, however, were considerably lower, ranging from .15 (Gv) to .26 (Glr). Thus, unit-weighted composite scores based on the six DAS-II CHC group factors possess insufficient construct-score variance³ for confident clinical interpretation (Reise, 2012; Reise et al., 2013).

Discussion

Because of the significant changes to the DAS-II structure, subtest content, and theoretical orientation, this study investigated the theoretical structure of the DAS-II with participants from the 5 to 8 year-old age range and sought to determine whether the theoretically-proposed six-factor CHC structure (i.e., Gc, Gf, Gwm, Glr, Gv and Gs) would emerge for this age group. Because participants within this age group were administered all 20 DAS-II subtests it was possible to test the theoretical alignment of the full test battery.

³ The argument that the group factors possess insufficient construct-score variance may not be consistent with the underlying theory of the DAS-II because g variance is included in the group-factor construct-score variance. Schneider (2013) offers an alternative for operationalizing construct-score variance that takes into consideration this theoretical distinction.

The present EFA-SL study extracted and examined three-, five-, and six-factor solutions. Although factor extraction decision-making rules recommended the extraction of up to five factors, six factors were extracted to be consistent with the proposed theoretical structure of the instrument and to examine performance of all six group factors. Generally, extraction of three or five factors as suggested by MAP, BIC and PA resulted in a solution that was less aligned (e.g., 5-factors) or struggled to align (e.g., 3-factors) with CHC theory. When five factors were extracted and examined using the SL solution (see Online Supplement Table A4) the results produced the following factors that inconsistently aligned with theoretically proposed factors: combined Gv/Gs, Gc, Gf, Glr, and Gwm. As noted, two subtests (i.e., Early Number Concepts [Gc/Gf] and Picture Similarities [Gf]) did not saliently load a group factor. Also, Gv and Gs formed a combined factor. A three-factor extraction (Online Supplement Table A3) produced factors and alignment of subtest loadings that were theoretically incoherent with the exception of a two-subtest Glr factor. These problems may indicate underextraction (Gorsuch, 1983; Wood, Tataryn, & Gorsuch, 1996).

When extracting six factors, the SL solution (Table 2) provided evidence for six plausible CHC group factors: Gc, Gv, Gf, Glr, Gs, and Gwm. This finding approached theoretical consistency where the expectation was for six distinct CHC group factors (see *Technical Handbook* and Keith et al., 2010). Furthermore, several of the factors (e.g., Gv, Glr, Gs, and Gwm) contained all of their theoretically proposed subtests. Two subtests, Picture Similarities (Gf) and Early Number Concepts (Gc/Gf), did not saliently load on a group factor causing the Gc and Gf factors in this study to diverge slightly from that proposed in the *Technical Handbook*.

⁴ Given that the Glr factor is defined by a single subtest offered twice (Recall of Objects-Immediate and Recall of Objects-Delayed) the emergence of the Glr factor was expected, if not guaranteed; therefore, the results may not provide the strongest evidence supporting the alignment the Glr factor with CHC theory in the DAS-II.

With the exception of these two subtests the results of this study were largely consistent with the CHC-based DAS-II structure posited in the *Technical Handbook*.

Nevertheless, the results of this study indicate that the DAS-II is primarily a measure of general intellectual ability. This conclusion is supported by the eigenvalues produced by principal axis factoring, the explained common and total variance produced by the SL analysis, and omega statistics. The preeminence of the general factor is a ubiquitous finding across most tests of cognitive ability that have been evaluated over the past decade (Canivez, 2016; Dombrowski & Watkins, 2016; Dombrowski, McGill & Canivez, 2017b).

Limitations and Future Directions

This study's results should be considered with the following limitations in mind. Most notably the 5 to 8 age range may represent an age range where the structure of the DAS-II may not generalize to older or younger age ranges (DiStefano & Dombrowski, 2006). However, as previously mentioned, Keith et al. (2010) found evidence for invariance of the DAS-II structure across the three age ranges presented in the *Technical Handbook* but needed to correlate several residuals (n=10), specify cross-loadings (n=10) and undertake a series of post hoc adjustments with a separate validation sample (n=5). Also, further research is needed to determine if the results of this study generalize to specific clinical (i.e., children referred for specific learning disability evaluations) or gifted populations.

Conclusion and Applied Clinical Implications

The results of this study provide evidence that across the 5 to 8 year old age range the DAS-II is a strong measure of general intellectual functioning and a tepid measure of six smaller CHC-related factors (Gc, Gf, Gwm, Gv, Gs, Glr). For instance, the results suggested that the general intelligence factor assumes 11 to 17 times more variance than the lower-order CHC

group factors. This finding is consistent with recent investigations of other instruments linked with CHC theory (e.g., WJ IV full test battery and WJ IV Cognitive [Dombrowski, McGill, & Canivez, 2017a, 2017b]; WJ III full test battery, WJ III Cognitive and WJ III Achievement [Dombrowski, 2013, 2014a, 2014b, 2015b; Dombrowski & Watkins, 2013]; WISC-V [Canivez, Watkins, & Dombrowski, 2016, 2017; Dombrowski, Canivez, & Watkins, 2017; Dombrowski, Beaujean, Canivez, & Watkins, 2015]; Stanford-Binet, Fifth Edition [Canivez, 2008; DiStefano & Dombrowski, 2006; Dombrowski, DiStefano, & Noonan, 2004]; and KABC-2 [McGill & Dombrowski, 2017; McGill & Spurgin, 2017]). The extant structural validity literature, along with the present study, suggests that although the lower-order group factors may be present and even theoretically consistent, they are of less significance in comparison with the general factor.

How should these results guide the interpretation of the DAS-II? First, except for two subtests, the DAS-II reflects well (and is essentially consistent with) the CHC-based theoretical structure posited in the *Technical Handbook*. On the surface this may augur positively for direct interpretation of the DAS-II indices. However, this practice must be considered against the backdrop of additional results. The nominal explained common and total variance of lower-order group factors and the low omega estimates suggest caution when engaging in DAS-II direct CHC-level interpretation and other interpretive heuristics such as cross-battery assessment (XBA) and processing strengths and weaknesses (PSW) analyses (e.g., Flanagan, Alfonso, & Mascolo, 2011; Flanagan, Ortiz, & Alfonoso, 2013; Niileksela, Reynolds, Keith & McGrew, 2016). Despite these possible clinical concerns the results of this study suggest that the DAS-II extended battery's structure at age 5 to 8 is generally consistent with the CHC-based theoretical alignment proposed in the *Technical Handbook*. The DAS-II is a measurement instrument that attains simple structure and generally reflects the six CHC factors (i.e., Gf, Gc, Gwm, Gv, Glr

and Gs) it proposes to measure. However, the dominance of general intelligence suggests the need for primary interpretive emphasis at that level.

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Table 1 DAS-II Factor Pattern (Structure) Coefficients from Principal Axis Factor Extraction with an Oblique (Promax) Rotation Age 5 to 8 (N = 787)

	Factor													
DAS-II Subtest (CHC factor)	I		II		III		IV		V		VI		h^2	u^2
Naming Vocabulary (Gc)	.79	(.76)	.13	(.52)	15	(.51)	01	(.36)	04	(.36)	.00	(.52)	.58	.42
Word Definitions (Gc)	.76	(.75)	24	(.41)	.13	(.56)	.05	(.38)	.04	(.36)	.02	(.51)	.59	.41
Verbal Similarities (Gc)	.68	(.77)	06	(.53)	.06	(.59)	.01	(.39)	.01	(.40)	.05	(.57)	.59	.41
Verbal Comprehension (Gc)	.52	(.67)	.17	(.54)	.09	(.57)	00	(.34)	08	(.34)	.03	(.53)	.47	.53
Picture Similarities (Gf)	.28	(.50)	.23	(.50)	.12	(.48)	04	(.26)	.10	(.39)	07	(.40)	.31	.69
Recall of Designs (Gv)	.05	(.52)	.91	(.82)	16	(.54)	.07	(.36)	01	(.49)	02	(.54)	.68	.32
Copying (Gv)	08	(.44)	.73	(.72)	04	(.52)	.01	(.33)	.02	(.46)	.08	(.51)	.52	.48
Matching Letter-Like Forms (Gv)	09	(.46)	.49	(.65)	.21	(.58)	.04	(.33)	10	(.36)	.16	(.55)	.46	.54
Pattern Construction-Alternative (Gv)	.15	(.61)	.43	(.72)	.31	(.70)	08	(.31)	.04	(.49)	04	(.56)	.59	.41
Recognition of Pictures (Gv)	00	(.43)	.38	(.57)	.27	(.53)	.17	(.38)	.04	(.40)	15	(.40)	.37	.63
Matrices (Gf)	.03	(.55)	07	(.52)	.77	(.74)	.04	(.33)	04	(.37)	.01	(.53)	.56	.44
Sequential and Quantitative Reasoning (Gf)	.10	(.63)	.17	(.67)	.68	(.81)	10	(.29)	04	(.43)	01	(.60)	.68	.32
Early Number Concepts (Gc/Gf)	.04	(.55)	.27	(.65)	.31	(.66)	.04	(.33)	.07	(.47)	.13	(.59)	.50	.50
Recall of Objects-Immediate (Glr)	.07	(.46)	01	(.41)	.04	(.40)	.81	(.84)	.02	(.39)	05	(.43)	.71	.29
Recall of Objects-Delayed (Glr)	03	(.37)	.05	(.36)	06	(.31)	.80	(.80)	03	(.32)	.06	(.40)	.64	.36
Speed of Information Processing (Gs)	02	(.32)	.00	(.43)	06	(.34)	.02	(.28)	.77	(.73)	.02	(.34)	.54	.46
Rapid Naming (Gs)	.01	(.37)	.22	(.48)	.02	(.40)	.07	(.32)	.30	(.50)	.04	(.39)	.30	.70
Digit Backward (Gwm)	06	(.55)	.06	(.60)	.21	(.65)	.00	(.39)	.04	(.44)	.58	(.75)	.59	.41
Digit Forward (Gwm)	.29	(.59)	.11	(.52)	19	(.48)	03	(.35)	01	(.37)	.54	(.67)	.49	.51
Recall of Sequential Order (Gwm)	.17	(.65)	04	(.58)	.20	(.66)	.09	(.46)	.01	(.44)	.44	(.73)	.59	.41
Eigenvalue	8.57		1.31		1.23		.89		.78		.75			
Variance (%)	42.4		6.5		6.1		4.5		3.9		3.8			
Factor Correlations														
Factor I	1.00													
Factor II	.65		1.00											
Factor III	.72		.74		1.00									
Factor IV	.48		.44		.41		1.00							
Factor V	.49		.62		.53		.42		1.00					
Factor VI	.69		.69		.72		.49		.50		1.00			

Note. h^2 = Communality coefficient. u^2 = Uniqueness, Gc = Comprehension-Knowledge, Gf = Fluid Reasoning, Gv = Visual Processing, Glr = Long-Term Storage and Retrieval, Gs = Perceptual Speed, Gwm = Working Memory. Pattern coefficients > .30 are **bolded** (Carroll, 1993, p. 108; Child, 2006). Subtest alignment with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicted following each subtest name.

Table 2 DAS-II Standardization Sample 5 to 8 Year Olds (N = 787) Sources of Variance According to a SL Orthogonalization with Six First—Order Factors

			Gc (Gv G		if G		lr	G	Gs		Gwm		
DAS-II Subtest	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	h^2
Naming Vocabulary	.65	.41	.42	.18	.07	.01	08	.01	01	.00	03	.00	.00	.00	.60
Word Definitions	.62	.39	.40	.16	13	.02	.08	.01	.03	.00	.03	.00	.02	.00	.57
Verbal Similarities	.67	.44	.36	.13	00	.00	.04	.00	.01	.00	.01	.00	.04	.00	.58
Verbal Comprehension	.62	.38	.27	.07	.09	.01	.05	.00	00	.00	06	.00	.03	.00	.47
Picture Similarities	.53	.28	.15	.02	.12	.02	.07	.01	02	.00	.08	.01	06	.00	.33
Recall of Designs	.68	.46	.03	.00	.48	.23	09	.01	.00	.00	01	.00	01	.00	.70
Copying	.59	.34	04	.00	.39	.15	02	.00	.01	.00	.01	.00	.07	.00	.50
Matching Letter-Like Forms	.57	.32	05	.00	.26	.07	.12	.01	.02	.00	08	.01	.14	.02	.43
Pattern Construction-															
Alternate	.69	.47	.08	.01	.23	.05	.18	.03	05	.00	.03	.00	03	.00	.56
Recognition of Pictures	.61	.38	00	.00	.20	.04	.15	.02	.10	.01	.03	.00	13	.02	.46
Matrices	.62	.38	.02	.00	04	.00	.40	.19	.02	.00	02	.00	.00	.00	.57
Sequential & Quantitative															
Reasoning	.68	.47	.05	.00	.09	.01	.38	.15	06	.00	03	.00	00	.00	.63
Early Number Concepts	.61	.38	.02	.00	.14	.02	.17	.03	02	.00	.05	.00	.11	.01	.44
Recall of Objects-															
Immediate	.73	.53	.04	.00	00	.00	.02	.00	.47	.22	.01	.00	04	.00	.76
Recall of Objects-Delayed	.63	.40	02	.00	.03	.00	04	.00	.47	.22	02	.00	.05	.00	.62
Speed of Information															
Processing	.43	.19	01	.00	.00	.00	03	.00	01	.00	.58	.34	.02	.00	.53
Rapid Naming	.48	.23	.01	.00	.11	.01	.01	.00	.04	.00	.23	.05	.03	.00	.30
Digit Backward	.52	.28	03	.00	.03	.00	.12	.01	.00	.00	.03	.00	.48	.23	.52
Digit Forward	.46	.21	.15	.02	.06	.00	10	.01	02	.00	01	.00	.45	.20	.46
Recall of Sequential Order	.61	.37	.09	.01	02	.00	.11	.01	.05	.00	.01	.00	.37	.14	.53
ECV (%)		69.0		6.0		6.0		5.0		4.0		4.0		6.0	.53
ETV (%)		37.0		3.0		3.0		3.0		3.0		3.0		3.0	
$\omega_{\rm H}/\omega_{\rm HS}$.88		.18		.15		.16		.26		.24		.24	

Note. Gc = Comprehension-Knowledge, Gf = Fluid Reasoning, Gv = Visual Processing, Glr = Long-Term Storage and Retrieval, Gs = Perceptual Speed, Gwm = Working Memory, h^2 = Communality, ECV = explained common variance, ETV = explained total variance, ω_H = omega-hierarchical, ω_{HS} = omega-hierarchical subscale.