

Exploratory and hierarchical factor analysis of the WJ-IV Cognitive at school age

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Abstract

Exploratory and confirmatory factor analytic studies were not reported in the Technical Manual for the Woodcock-Johnson, Fourth Edition Cognitive (WJ IV Cognitive; Schrank, McGrew, & Mather, 2014b). Instead, the internal structure of the WJ IV Cognitive was extrapolated from analyses based on the full WJ IV test battery (Schrank, McGrew & Mather, 2014b). Even if the veracity of extrapolating from the WJ IV full battery were accepted, there were shortcomings in the choices of analyses used and only limited information regarding those analyses was presented in the WJ IV *Technical Manual* (McGrew, Laforce, & Schrank, 2014). The present study examined the structure of the WJ IV Cognitive using EFA procedures [principal axis factoring with oblique (promax) rotation followed by application of the Schmid-Leiman (1957) procedure] applied to standardization sample correlation matrices for two school age groups (ages 9-13; 14-19). Four factors emerged for both the 9-13 and 14-19 age groups in contrast to the publisher's proposed 7 factors. Results of these analyses indicated a robust manifestation of general intelligence (*g*) that exceeded the variance attributed to the lower-order factors. Model-based reliability estimates supported interpretation of the higher-order factor (i.e., *g*). Additional analyses were conducted by forcing extraction of the seven theoretically posited factors; however, the resulting solution was only partially aligned (i.e., *Gs*, *Gwm*) with the theoretical structure promoted in the *Technical Manual* and suggested the preeminence of the higher-order factor. Results challenge the hypothesized structure of the WJ IV Cognitive and raise concerns about its alignment with Cattell-Horn-Carroll theory.

KEYWORDS: Exploratory Factor Analysis; Higher-order Factor Analysis; Cattell-Horn-Carroll Theory; Schmid-Leiman Orthogonalization; General Intelligence

Exploratory and Hierarchical Factor Analysis of the WJ-IV Cognitive at School Age

The Woodcock-Johnson IV (WJ IV; Schrank, McGrew & Mather, 2014a) was recently revised and comprises separate co-normed tests of cognitive ability, achievement, and oral language (Schrank, McGrew & Mather, 2014b; Schrank, Mather & McGrew, 2014a, b). The WJ IV authors report that the instrument's structure was guided by Carroll's (1993) Three Stratum Theory of Cognitive Ability, the work of Horn and Cattell (1966) along with contemporary neuroscience research on memory (e.g., McGrew, LaForte & Schrank, 2014). The *Technical Manual* indicates that the WJ IV Cognitive was designed to measure a hierarchically ordered general intellectual ability factor (i.e., g) along with the lower-order factors of Comprehension-Knowledge (Gc), Fluid Reasoning (Gf), Short-Term Working Memory (Gwm), Cognitive Processing Speed (Gs), Auditory Processing (Ga), Long Term Retrieval (Gltr), Visual-Processing (Gv) (McGrew et al., 2014).

Of concern, the WJ IV Cognitive was never separately subjected to exploratory or confirmatory factor analytic procedures (or if such analyses were undertaken they were not reported). Instead, the internal structure of the WJ IV Cognitive was extrapolated from analyses based on the full WJ IV test battery (Schrank, McGrew & Mather, 2014b). Even if one were to accept the veracity of simply extrapolating the WJ IV Cognitive structure from that of the full WJ IV test battery, considerable problems would remain. One of these concerns relates to the choice of exploratory factor analysis (EFA).

When reviewing the literature for the development of the WJ IV, the test authors overlooked prior, relevant exploratory structural validity and predictive validity research on the Woodcock-Johnson, Third Edition (WJ III). Several of these studies specifically analyzed the WJ III Cognitive (e.g., Dombrowski, 2014a; Dombrowski, 2014b; McGill, 2015; McGill &

Busse, 2015). The conclusion of these structural validity studies was that the WJ III Cognitive was overfactored (i.e., too many factors extracted), was a solid measure of general intellectual ability, but caution should be heeded when interpreting scores beyond the higher-order factor. Dombrowski (2014a; 2014b) reported that the WJ III Cognitive was a 4-factor test for ages 9 to 13 and a 3-factor test for ages 14 to 19. A related study investigated the internal structure of the full WJ III test battery using principal axis factoring (PAF) with promax rotation followed by the Schmid-Leiman procedure and found that the structure posited in the WJ III *Technical Manual* yielded Heywood cases, impermissible factors because of no salient loadings, and a lack of convergence with publisher theory (Dombrowski & Watkins, 2013).

When analyzing the structure of the full WJ IV battery using principal axis factoring (PAF) with an oblique rotation McGrew et al. (2014) produced similar results as Dombrowski and Watkins (2013) finding Heywood cases, impermissible factors, and a lack of convergence of the full test battery structure. Instead of attributing these results to over-extraction, which can cause these problems (e.g., Gorsuch, 1983; Harman, 1976; Thompson, 2004), McGrew et al. (2014a) conducted a different, albeit technically less appropriate method of analysis [i.e., principal components analysis (PCA) with varimax rotation]. However, McGrew et al. (2014) did not furnish the results of their PAF analysis so one cannot determine whether there was any practical difference in the patterns of loadings between the two analyses (i.e., PCA or PAF).

Regardless, some have argued that PCA is not a factor analytic procedure (Fabrigar & Wegener, 2012; Gorsuch, 1983). Osborne (2015) elaborated, “It is also not considered a true method of factor analysis and there is disagreement among statisticians about when it should be used, if at all” (p. 1). PCA is a computationally simplified version of the general class of dimension reduction analyses. It computes the analysis without consideration of the underlying

latent structure of the variables, using all the variance in the manifest variables, and therefore does not discriminate between different dimensions of variance (e.g., shared and unique variance). Accordingly, the components derived from PCA should not be interpreted as a reflection of latent dimensions such as a general factor (Bentler & Kano, 1990; Jensen, 1998; Preacher & MacCallum, 2003; Widaman, 1993). Instead, the measured variables in PCA are of interest rather than a hypothetical latent construct such as general intelligence or group factors (i.e. visual-spatial processing; crystallized ability). Widaman (1993) observed that salient loadings are higher in PCA than in factor analysis and that such inflation is magnified when the salient loadings are more moderate in value (e.g., 0.40 in the population) rather than high (e.g., 0.80). Commenting on the accuracy of PCA v. PAF, Fabrigar et al (1999) noted that PAF removes random error from the factors so the relation among factors in a PAF analysis are more likely to approach the population values. Further, PAF includes only common variance. PCA includes common and specific variance, which can inflate factor loadings and give the specious appearance of a stronger factor structure (Snook & Gorsuch, 1989). Constituting a further possible problem with the EFA analyses undertaken, McGrew et al. (2014a) used a type of rotation (e.g., varimax) following factor extraction that some consider inappropriate. Gorsuch (1983) commented that “varimax is inappropriate if the theoretical expectation suggests a general factor may occur” (p. 185). With the genesis of Carroll’s Three Stratum Theory the general factor is now generally accepted (Beaujean, 2015). Varimax rotation is also considered inappropriate when factors are highly correlated as in the case of tests of cognitive ability such as the WJ IV Cognitive. Consequently, in these circumstances, an oblique rotation (e.g., promax) is considered appropriate and necessary (Thompson, 2004). Although an oblique rotation is necessary, it is not singularly sufficient and an additional step is required. Gorsuch (1983)

commented that higher-order factors are implicit in all oblique rotations, so it is recommended that these factors be extracted and examined (Gorsuch, 1983). McGrew et al. (2014a) did not include this additional step because an orthogonal rotation such as varimax does not produce correlations between rotated factors, and therefore eliminates the option of undertaking a second-order factor analysis.

One approach to examining and extracting higher-order factors is through the Schmid-Leiman (SL; 1957) procedure (Carretta & Ree, 2001; Carroll, 1993, 1995, 2003; Gustafsson & Snow, 1997). This procedure was used by Carroll (1993) when he created his Three Stratum Theory of Cognitive Abilities. It is surprising that the SL procedure was overlooked when developing the WJ IV because Carroll's Three Stratum Theory was cited as being highly influential in the development of CHC and the WJ IV. It is also noteworthy that Dombrowski and colleagues (e.g., Dombrowski & Watkins, 2013; Dombrowski, 2014a, 2014b; 2015a) highlighted this critical omission in a series of WJ III articles. Further, the SL procedure has been used to examine the structure of numerous other tests of cognitive abilities including the Cognitive Assessment System (e.g., Canivez, 2011), the Reynolds Intellectual Assessment Scales (e.g., Dombrowski, Watkins & Brogan, 2009; Nelson & Canivez, 2012), the Stanford-Binet, Fifth Edition (e.g., Canivez, 2008; DiStefano & Dombrowski, 2006), and Wechsler scales (e.g., Canivez, 2014; Canivez & Watkins, 2010 a, b; 2014; Canivez & Kush, 2013; Canivez, Watkins & Dombrowski, 2015). Finally, McGrew (2012) indicated that he conducted the SL procedure on the 50 subtest WJ III battery but the results of those analyses have never been published or presented in any form.

The SL procedure involves making first-order factors orthogonal to second-order factors by first extracting the variance explained by the second-order factors. The next step in the

procedure is to residualize the first-order factors of all the variance present in the second order factors. Schmid and Leiman (1957) argued that this process “preserves the desired characteristics of the oblique solution” and “discloses the hierarchical structure of the variables” (p. 53). Carroll (1995) emphasized that “...orthogonal factors should be those produced by the Schmid-Leiman (1957) orthogonalization procedure...” (Carroll, 1995, p. 437) noting in fact that this transformation produces “...an orthogonal factor pattern very similar to the Spearman-Holinger bi-factor pattern” (Carroll, 1990, p. 90). The SL procedure permits variance partitioning (i.e., determining the variance accounted for by higher and lower order factors), which assists when attempting to make clinical interpretation decisions. The SL procedure was used to examine the structure of the WJ III Cognitive, Achievement, and full test battery (Dombrowski, 2014a, 2014b, 2015b; Dombrowski & Watkins, 2013). Recently, Jennrich and Bentler (2011) developed a procedure that reveals exploratory bifactor structure (EFA with a bifactor rotation). This procedure and others (e.g., Bayesian Structural Equation Modeling) may also be tenable alternatives for exploring internal structure.

There are additional problems with the exploratory factor analyses presented in the WJ IV *Technical Manual*. McGrew et al. (2014) did not present rudimentary factor analytic statistics including the correlation among factors from their EFA/oblique analysis, percentage of variance accounted for by higher- and lower-order factors, communality estimates, and model based reliability estimates including omega-hierarchical (ω_H) and omega-hierarchical subscale (ω_{HS}) (Canivez, in press a; Reise, 2012, Rodriguez, Reise, & Haviland, 2015). The corpus of literature on EFA methodology (e.g., Carroll, 1993, 1995, 1997, 2003; Gorsuch, 1983; McClain, 1996; Ree, Carretta and Green, 2003; Thompson, 2004) and model-based reliability (e.g., Reise, 2012; Reise, Bonifay, & Haviland, 2013; Rodriguez et al., 2015) recommends the inclusion of

this information because it aides test users in determining how the instrument should be interpreted.

Along with problems with the EFA analyses used, there were also problems associated with the confirmatory factor analytic procedures reported. First, the WJ IV *Technical Manual* reported only testing a few competing confirmatory factor analysis (CFA) models (Model 1: single *g* factor; Model 2: 9 broad CHC higher-order model; Model 3: broad plus narrow CHC higher-order factor model). It is also unknown why rival, plausible models such as Woodcock's Cognitive Performance Model (e.g., Taub & McGrew, 2014) model or Dombrowski & Watkin's (2013) Schmid-Leiman models were not considered. Second, best fitting initial models and cross-validation models across each age group had comparative fit indexes (CFI; .603–.700), Tucker–Lewis Indexes (TLI; .607–.684), or root mean square errors of approximation (RMSEA; .115–.123) that did not approach levels considered to be adequate (CFI, TLI \geq .90; Hu & Bentler, 1999; RMSEA \leq .08; Hu & Bentler, 1999). Thus, the WJ IV structural models tested were not well-fitting. Third, the WJ IV data were *multivariate non-normal* (Mardia's [1970] multivariate kurtosis estimate was 27.6) and produced Heywood cases. Consequently, maximum likelihood (ML) estimates should not be relied upon and robust ML estimates should instead be used (Byrne, 2006, 2012). The *Mplus* MLM (robust estimator) produced the following estimates for the final age 9 to 13 MD model: CFI = .638; TLI = .610, and RMSEA = .116. These results were quite close to the ML estimates previously reported and indicated poor model fit. Reported *Mplus* MLMV estimates were equally poor and pose serious questions about the merits of the proposed WJ IV structure (see Canivez, in press b for additional details).

Although McGrew et al.'s (2014) claim that they “left no stone unturned” (p. 179) is laudable, the scholarly evidence presented in the WJ IV *Technical Manual* does not live up to

this sentiment. The omission of a separate examination of the structure of the WJ IV Cognitive, the less-than-optimal choice of EFA methodology, the lack of inclusion of rudimentary EFA statistics, the omission of model-based reliability estimates, the omission of variance partitioning, the poor CFA results, and the incomplete review of the WJ III structural validity literature suggests that field's understanding the structure of the WJ IV Cognitive is far from complete. This study aims to overcome some of these shortcomings by investigating the internal structure of the WJ IV Cognitive during school age (e.g., ages 9 to 19) using exploratory and higher-order exploratory factor analyses.

Method

Participants

The WJ IV *Technical Manual* reported information relative to eight age groups: 2 to 3 years; 4 to 5 years, 6 to 8 years, 9 to 13 years, 14 to 19 years, 20 to 29 years, 30 to 39 years, and 40 years and older. Data for the WJ IV norms were collected from a nationally representative sample of 7,416 participants from age 2 through 90 plus. The WJ IV *Technical Manual* reports that the normative data controlled for census region, sex, country of birth, race, community type, parent education, and occupational level. Detailed demographic characteristics are provided in the WJ IV *Technical Manual*. For this study, two school aged (9 to 13 years and 14 to 19 years) subtest correlation matrices (18 by 18) were obtained from the *Technical Manual*. The 9-13 age group contained an average of 1,582 participants whereas the 14-19 age group contained an average of 1,685 participants.

Instrument

The Woodcock-Johnson IV Tests of Cognitive Abilities (WJ IV Cognitive; Schrank, McGrew & Mather, 2014b) contains 18 cognitive tests that are purported to measure *g* and seven

Cattell-Horn-Carroll (CHC) factors: visual-spatial thinking (Gv), fluid reasoning (Gf), processing speed (Gs), long-term retrieval (Glr), auditory processing (Ga), short-term memory (Gwm), and comprehension-knowledge (Gc). The WJ IV Cognitive also yields a general intellectual ability score reflective of *g*. From a theoretical perspective, the authors of the WJ IV reported that they were guided by Carroll's (1993) Three Stratum Theory of Cognitive Abilities, the work of Horn and Cattell (1966), and recent neuroscience-based memory research which served as the basis for Cattell-Horn-Carroll (CHC; McGrew & Woodcock, 2001) theory. It should be noted that the WJ IV Cognitive remains the only cotemporary intelligence test that purports to measure all of the broad abilities associated with the latest iteration of the CHC model (Schneider & McGrew, 2012). Within the *Technical Manual*, 18 subtest correlation matrices across eight different age ranges (2 to 3; 4 to 5; 6 to 8; 9 to 13; 14 to 19; 20 to 39; 40 plus) were included to show the correlation among cognitive subtests (McGrew, LaForte & Schrank, 2014). The WJ IV *Technical Manual* presents the relationships of the WJ IV Cognitive subtests with their relationship to *g* and seven Cattell-Horn-Carroll (CHC) factors.

Procedure and Analyses

The correlation matrices within this study were analyzed using several EFA methods with SPSS v. 23 and additional software. First, the intercorrelation matrices for the two age groups (9-13, 14-19) were evaluated using Bartlett's Test of Sphericity (Bartlett, 1954) and the Kaiser-Meyer-Olkin (KMO; Kaiser, 1974) statistic to ensure that the matrices were suitable for factor analysis. Second, the intercorrelation matrices were subjected to principal axis factoring (Cudeck, 2000; Fabrigar, Wegener, MacCallum & Strahan, 1999; Tabachnick & Fidell, 2007) with promax rotation ($k = 4$; Tataryn, Wood, & Gorsuch, 1999) because of the assumption of correlated factors (Gorsuch, 1983; Schmitt, 2011; Tabachnik & Fidell, 2007). Factor pattern

coefficients $\geq .30$ were considered salient (Child, 2006; Schmitt, 2011). Multiple factor extraction criteria were examined (Gorsuch, 1983) as well as factor interpretability and compliance with simple structure (Thurstone, 1947). Specifically, the scree test (Cattell, 1966), standard error of scree (SE_{Scree} ; Zoski & Jurs, 1996), Horn's parallel analysis (HPA; Horn, 1965), and minimum average partials (MAP; Velicer, 1976) were examined. These HPA and MAP procedures were conducted using O'Conner's (2000) SPSS syntax while SE_{Scree} (Watkins, 2007) was used because it was reportedly the most accurate objective scree method (Nasser, Benson, & Wisenbaker, 2002). Higher-order factor analysis of the promax rotated factor correlations and use of the Schmid–Leiman (1957) procedure was applied to the oblique first-order factors to elucidate the structure of the WJ IV Cognitive using the SPSS syntax furnished by Wolff and Preising (2005). Model-based reliability estimates (i.e., Omega estimates; ω_H and ω_{HS}) were produced using the *Omega* program developed by Watkins (2013). It has been suggested that omega coefficients should at a minimum exceed .50, but .75 is preferred (Reise, 2012; Reise, Bonifay, & Haviland, 2013).

Results

Bartlett's Test of Sphericity (Bartlett, 1950) for both age group analyses indicated that the correlation matrices were not random (9 to 13 age range $\chi^2 = 11.592.57$, $df = 153$, $p < .0001$; 14 to 19 age range $\chi^2 = 13,819.03$, $df = 153$, $p < .0001$). For the 9-13 and 14-19 age groups, the Kaiser–Meyer–Olkin (Kaiser, 1974) statistic was .867 and .894, respectively, well above the minimum standard for conducting a factor analysis suggested by Kline (1994). Measures of sampling adequacy for each variable were also within reasonable limits. Thus, the correlation matrices were deemed appropriate for factor analysis.

Factor Extraction Criteria Comparisons

Parallel analysis (Horn, 1965) suggested that three, almost four, factors be retained for the 9-13 age group, while four factors were indicated for the 14-19 age group (See Figures 1 & 2). SE_{Scree} indicated that there were three non-trivial factors for the 9-13 age group and four non-trivial factors for the 14-19 age group. The MAP (Velicer, 1976) criterion recommended retention of two factors for the 9-13 age group and one factor for the 14-19 age group. Visual examination of scree plots indicated evidence for four factors for each age group. The extraction of four factors at both age ranges made most sense, but interpretability and linkage to theory was still problematic when attempting to interpret beyond the general factor. While none of the factor extraction criteria suggested seven latent factors examination of results from the forced extraction of seven factors allowed for inspection of subtest alignment with theoretically proposed factors as well as performance of smaller factors.

Seven-Factor Exploratory and Hierarchical Analyses

Tables 1 and 2 present the results of the PAF analyses with oblique (promax) rotation for the age 9-13 age group and the 14-19 age group correlation matrices, respectively, in accord with a seven factor extraction. First unrotated factor coefficients (*g* loadings) ranged from .44 to .69 for the 9-13 age group and from .38 to .74 for the 14-19 age group. Based on Kaufman's (1994) criteria ($\geq .70$ = good, $.50 - .69$ = fair, $< .50$ = poor) three of the age 9-13 subtests (Picture Recognition, Visual-Auditory Learning and Pair Cancellation) had poor *g* loadings while the remaining age 9-13 subtests were in the fair range. For the 14-19 age group two subtests (Picture Recognition and Visual-Auditory Learning) displayed poor *g* loadings while Object Number Sequence and Oral Vocabulary were in the good range. The remaining subtests were in the fair range based on Kaufman's (1994) criteria ($\geq .70$ = good, $.50 - .69$ = fair, $< .50$ = poor). Analyses for the 9-13 and 14-19 age groups indicated that the first factor accounted for 36.51% and

36.96% of the variance, respectively. This dwarfed the variance accounted for by the second factor in both the 9-13 and 14-19 age groups (9.15% and 8.03%, respectively). For the 9-13 age group, factors five through seven were essentially trivial factors with nominal root sizes. Also, for this age group Factors VI and VII possessed only one salient subtest and were judged inadequate. Other problems included subtest migration to theoretically different factors as well as subtests loading saliently on more than one factor (cross-loading). For the 14-19 age group, three of the seven factors were essentially trivial factors with nominal root sizes. Factor VII was inadequate with no salient subtest loadings and subtest migration to theoretically different factors were also observed. Correlations among the seven extracted factors ranged from .09 to .61 (*Mdn* = .34) for the 9-13 age group. Correlations among the seven extracted factors ranged from .03 to .65 (*Mdn* = .475) for the 14-19 age group. These high correlations among factors implies the presence of a higher-order factor, which needed to be extracted and examined (Gorsuch, 1983; Thompson, 2004).

Results from the Schmid and Leiman (1957) orthogonalization procedure of the seven factor solutions for both age groups are presented in Tables 3 (9-13 age group) and 4 (14-19 age group). In the age 9-13 age group Schmid-Leiman (SL) analysis, the higher-order factor (*g*) accounted for 27% of the total variance and 55% of the common variance. In the age 14-19 age group SL analysis, the higher-order factor (*g*) accounted for 30% of the total variance and 59% of the common variance. The general factor also accounted for between 17% and 44% (*Mdn* = 29.5%) of individual subtest variance in the 9-13 age group. The *g* factor accounted for between 15% and 51% (*Mdn* = 32%) of individual subtest variability in the 14-19 age group. For the 9-13 age group, the seven first-order group factors accounted for small proportions of the total variance (2% to 4%) and common variance (3.2% to 8.6%). The first- and second-order factors

combined to measure 54.0% of the variance in the WJ IV Cognitive, reflecting 46.0% unique variance. For the 14-19 age group, the seven, first-order group factors accounted for 1% to 4% of the total variance and 2% to 8% of the common variance. The first- and second-order factors of the 14-19 age group combined to measure 56% of the variance in the WJ IV Cognitive, reflecting 44% unique variance. Results of both analyses demonstrated robust manifestation of general intelligence in the WJ IV Cognitive where the combined influence of general intelligence and uniqueness exceeded the contributions made by the first-order factors.

The model-based reliabilities of the WJ IV were estimated with ω_H and ω_{HS} . The ω_H coefficient for the general factor was high for both age groups (.83 and .85) and sufficient for interpretation. The ω_{HS} coefficients ranged from .11 to .42 in the 9-13 age group and .08 to .31 in the 14-19 age group, which appear considerably lower than the minimum suggested for interpretation (Reise, 2012; Reise et al., 2013).

Four-Factor Exploratory and Hierarchical Analyses

Ages 9-13 First-Order EFA: Four Factor Extraction. Table 5 presents detailed information regarding EFA and g loadings ranged from .43 to .69, all of which were in the fair range with the exception of Pair Cancellation, Picture Recognition, and Visual-Auditory Learning which were in the poor range. Factor I combined all the Gwm and Ga subtests in addition to Memory for Words, an Auditory Memory subtest. All the Gv and Glr subtests along with two of the Gf (Analysis-Synthesis, Concept Formation) subtests combined as indicators of Factor II; Gs, Perceptual Speed, and Number Series (Gf) subtests combined as indicators of Factor III; and the Gc subtests combined as indicators of Factor IV. Simple structure was observed with no subtest cross-loading. The four factors were moderately correlated, implying a higher-order factor requiring explication.

Ages 9-13 SL Hierarchical Analyses: Four First-Order Factors. Table 6 presents results from the Schmid and Leiman orthogonalization of the higher-order factor analysis. The general factor accounted for 61.0% of the common variance and 23.8% of the total variance, exceeding that accounted for by the lower order factors (7.9% to 12.0% common variance; 3.3% to 7.2% total variance). The first- and second-order factors combined to measure 47.9% of the variance in the WJ IV Cognitive, reflecting 52.1% unique variance. Results demonstrated a robust manifestation of general intelligence in the WJ IV where the combined influence of general intelligence and uniqueness exceeded the contributions made by the first-order factors. The ω_H coefficient for the general factor was high (.797) and sufficient for interpretation. Omega-hierarchical subscale coefficients for the group factors ranged from .213 to .489. All of the ω_{HS} estimates, save Factor IV (Gc), were considerably lower than the minimum suggested for interpretation.

Ages 14-19 First-Order EFA: Four Factor Extraction. Table 7 presents detailed information regarding EFA and g loadings ranged from .37 to .74, all of which were in the fair range except for Picture Recognition and Visual-Auditory Learning which were in the poor range based upon Kaufman's (1994) criteria ($\geq .70$ = good, $.50 - .69$ = fair, $< .50$ = poor). Object Number Sequence and Oral Vocabulary were in the good range. All the Gv and Glr subtests combined with two of the Gf (Analysis-Synthesis, Concept Formation) subtests as indicators of Factor I; the Gs subtests and the Perceptual Speed subtest combined as an indicator of Factor II; Gwm, Auditory Memory and Nonword Repetition, a Ga subtest as indicators of Factor III; and Gc subtests along with Phonological Processing (Ga) and Number Series (Gf) combined as indicators of Factor IV. Simple structure was observed with no subtest cross-loading. The four factors were moderately correlated, implying a higher-order factor requiring explication.

Ages 9–13 SL Hierarchical Analyses: Four First–Order Factors. Table 8 presents results from the Schmid and Leiman orthogonalization of the higher–order factor analysis. The general factor accounted for 64.3% of the common variance and 29.1% of the total variance, dwarfing that apportioned to lower order factors (7.7% to 10.3% common variance; 3.5% to 4.7% total variance). The first- and second-order factors combined to measure 50.3% of the variance in the WJ IV Cognitive, reflecting 49.7% unique variance. Results demonstrated a robust manifestation of general intelligence in the WJ IV Cognitive where the combined influence of general intelligence and uniqueness exceeded the contributions made by the first-order factors. The ω_H coefficient for the general factor was high (.805) and sufficient for interpretation. The ω_{HS} coefficients for the group factors ranged from .203 to .336 and were considerably lower than the minimum suggested for interpretation.

Discussion

McGrew et al. (2014) did not report a separate examination of the structure of the WJ IV Cognitive. Instead they extrapolated the WJ IV Cognitive structure from an analysis of the full WJ IV battery. Thus, the structure of the WJ IV Cognitive is not fully understood. Extrapolation of the results from the WJ IV full battery yielded 7 factors for the WJ IV Cognitive. However, there were considerable problems with choices within both EFA and CFA when analyzing the full WJ IV battery (McGrew et al., 2014).

First, McGrew et al. (2014) examined the structure of the WJ IV full battery using two EFA procedures (PCA with varimax rotation and PAF with promax rotation) but did not present the PAF/promax results, claiming that they yielded Heywood cases and a lack of model convergence. These findings may likely be the result of overfactoring (e.g., Gorsuch, 1983; Harman, 1976; Thompson, 2004) McGrew et al.). And, the use of PCA with varimax rotation

has been criticized as not even a type of factor analysis and possibly inappropriate for tests of cognitive ability such as the WJ IV (Gorsuch, 1983; Osborne, 2015).

Additional problems with the McGrew et al. (2014) analyses were that they did not include rudimentary EFA statistics including eigenvalues, percent of variance attributed to higher- and lower-order factors, communality statistics, the correlation among the factors, and variance partitioning. Of note, Dombrowski and colleagues (2013, 2014a, 2014b, 2015a) pointed out these omissions in a series of articles on the prior version of the WJ. Variance partitioning (i.e., ascription of variance to higher and lower order factors) may be attained by using the SL procedure (Schmid & Leiman, 1957). This procedure was explicitly recommended by Carroll and other experts in factor analysis (e.g., Carretta & Ree, 2001; Carroll, 1993, 1995, 2003; Gorsuch, 1983; Gustafsson & Snow, 1997; McClain, 1996; Ree, Carretta and Green, 2003; Thompson, 2004) and used by Carroll to arrive at his Three Stratum Theory. Regarding the omission of the SL procedure, this is no small oversight as CHC theory was predicated in part upon Carroll's theory which itself was derived through the use of the SL procedure. Inexplicably, McGrew (2012) undertook an analysis of the 50 subtest WJ-III using the SL procedure but has yet to publish the details of those analyses. Finally, there were problems with McGrew et al.'s CFA methods used to investigate the WJ-IV. They examined only three competing models, the results of which showed poor model fit on several fit statistics (i.e., RMSEA, CFI, TLI).

Because of these important evidentiary, theoretical, and psychometric omissions, the two school aged (9-13 years; 14-19 years) correlation matrices were subjected to EFA and higher-order factor analysis. Use of EFA factor extraction procedures (e.g., parallel analysis, SE_{Scree} , and MAP as supplemented by a visual scree) that are considered to be psychometrically robust suggests that the WJ IV Cognitive is at best a four factor test for both age groups. Extracting this

number of factors indicated that the WJ IV Cognitive is a solid measure of psychometric *g* but fails to align with the posited test structure (CHC). Lower-order factors and subtests were problematic because they lacked full linkage to theory and the structure posited in the WJ IV *Technical Manual*.

For the 9-13 age group (Table 5), extraction of four factors suggests that three of the factors cohere with the structure posited in the WJ IV *Technical Manual*: Gwm (Verbal Attention, Memory for Words, and Object Naming); Gs (Letter-Pattern Matching; Number-Pattern Matching and PC); and Gc (Oral Vocabulary and General Information). A fourth factor is reminiscent of the perceptual reasoning factor from the Wechsler Scales, which combined four subtests: Visual-Spatial, Visual-Auditory Learning, Picture Recognition and Analysis-Synthesis. According to a SL orthogonalization extracting four factors in this age range suggested that 6 subtests (e.g., Nonword Repetition, Phonological Processing, Numbers Reversed, Concept Formation, and Story Recall) did not contain salient residual loadings on any group factor.

For the 14-19 age group (Table 7), extraction of four factors yielded a structure roughly similar to the 9-13 age range but divergent from that posited in the WJ IV *Technical Manual*. The second factor measures processing speed (Gs): Number-Pattern Matching, Letter-Pattern Matching and Pair Cancellation. The third factor appears to be a working memory factor (Gwm): Verbal Attention, Object-Number Sequencing, and Memory for Words along with Nonword Repetition (Ga), an auditory processing subtest. The fourth factor is reminiscent of crystallized ability (i.e., Gc): Oral Vocabulary and General Information plus Phonological Processing. A perceptual reasoning factor also emerged through a combination of subtests on the first factor [e.g., Visualization (Gv), Picture Recognition (Gv) and Analysis-Synthesis (Gf)]. The remaining subtests did not load any group factor. The result of the analyses across the 9 to 19 age range

suggests an inability to locate distinct Gf, Gv, and Ga factors. The only consistent finding across both age ranges was for a processing speed factor that cohered with the structure posited in the WJ IV *Technical Manual*. Whereas the PAF/promax analysis resulted in simple structure with all subtests loading saliently on a group factor with no cross loading, the SL analysis, which removed the influence of *g*, indicated that several of the subtests across both age groups no longer had salient residual group factor loadings. This result is not surprising given the pervasive influence of *g* in an instrument such as the WJ IV Cognitive.

EFA procedures within this study did not identify the Gv, Gf, Gv, and Ga factors and therefore did not support the seven factor model posited in the WJ IV *Technical Manual*. Instead, structural analyses using psychometrically sound EFA procedures indicated that the WJ IV Cognitive is a solid measure of general intelligence. There is also some evidence that the WJ IV Cognitive may provide a measure of Gs, Gwm and Gc across both age groups plus a factor reminiscent of perceptual reasoning. However, the structure of those factors comprises different subtests than what is suggested in the *Technical Manual*. Model-based reliability estimates also were insufficiently high for confident independent interpretation of lower-order (i.e., index level) factors as they contain too little unique true score variance (Reise et al., 2013).

When disregarding factor extraction decision rules and extracting seven factors in accord with the recommended extraction posited in the *Technical Manual*, the WJ IV structure was not supported as hypothesized. As noted in Tables 3 and 4, for both age groups, the subtests that load Gc (General Information, Oral Vocabulary) and Gs (Pair Cancellation, Letter-Pattern Matching and Number-Pattern Matching) were aligned with their theoretically consistent CHC factors. Beyond that alignment the structure for both age groups diverged from that presented in the *Technical Manual*.

For the 9-13 age group, Verbal Attention (Gwm), Memory for Words (Auditory Memory), Object-Number Sequence (Gwm), and Nonword Repetition (Ga) combined to form a working memory and auditory processing (Gwm/Ga) factor. Picture Recognition (Gv), Visualization (Gv), Visual-Auditory Learning (Glr) and Analysis-Synthesis (Gf) combine to form a second factor (i.e., perceptual reasoning). Factors III and IV form the aforementioned 9-13 age group Gs and Gc factors. Factors IV through VII are impermissible because of a single subtest loading.

For the 14-19 age group, Verbal Attention (Gwm), Memory for Words (Auditory Memory Span), Nonword Repetition (Ga) and Object-Number Sequence (Gwm) combined to form a Gwm/Ga factor. This factor is the same as the one formed for the 9-13 age group. Factors II and III combined the same subtests as the 9-13 age group to form Gs and Gc factors. Factor IV was trivial because of a single salient subtest loading. Factor V comprised two of the three Gf subtests (Number Series and Analysis-Synthesis) but did not capture Concept Formation (Gf). Instead, Concept-Formation (Gf) and Visual-Auditory Learning (Glr) combined to form the sixth factor. The seventh factor for the 14-19 age group did not contain any salient loadings and therefore was not viable. In sum, for both age groups when attempting to force the seven factor solution by discarding the most lenient factor extraction decision rules, the WJ IV Cognitive structure does not hold.

Conclusion

The results of this study revealed that the WJ IV Cognitive is primarily a measure of *g*, as it accounts for a majority of the subtests' total and common variance. The preeminence of *g* found in this study is similar to the findings of other studies of intelligence tests using both EFA and CFA methods (Bodin et al., 2009; Canivez, 2008; Canivez et al., 2009; Canivez, 2011;

Canivez, 2014b; Canivez & Watkins, 2010a, 2010b; Canivez, Watkins & Watkins, 2015; DiStefano & Dombrowski, 2006; Dombrowski, 2013, 2014a, 2014b; 2015b; Dombrowski et al., 2009; Dombrowski & Watkins, 2013; Dombrowski, Canivez & Watkins, 2015; Gignac & Watkins, 2013; Nelson & Canivez, 2012; Nelson, et al., 2007; Nelson et al., 2013; Watkins, 2006; 2010; Watkins & Beaujean, 2014; Watkins et al., 2013; Watkins et al., 2006). Similarly, these results are consistent with the broader professional literature on the importance of general intelligence (Deary, 2013; Jensen, 1998; Lubinski, 2000; Ree et al., 2003). Additional broad-contextual research is necessary which studies the development of specific abilities over time and examines the prediction of meaningful outcomes (clinical, educational, home/family) and relative/differential prediction by age group. It may also be worthwhile to investigate the extent to which the factor structure is equivalent/invariant across age groups as this has not been formally examined within the present study.

Considering that most of the WJ IV Cognitive variance was contributed by g and that ω_{HS} coefficients were low, primary interpretive emphasis should be placed upon the general factor as manifested in the General Intellectual Ability score. In contrast with the numerous interpretive claims made in the WJ IV *Technical Manual*, the results of this study provide little empirical justification for the clinical interpretation of group factors or their manifestations in the variety of index scores and comparisons that can be calculated from the WJ IV Cognitive subtests (Reise, 2012; Reise et al., 2013; Reise, Moore & Haviland, 2010; Rodriguez et al., 2015). Furthermore, the lack of evidence to support several of the Stratum II dimensions posited in the CHC-based seven factor model suggest that interpretation of these indicators within clinical practice may be problematic. This is consistent with the findings from two predictive validity studies on the WJ III Cognitive (McGill, 2015; McGill & Busse, 2015).

The *Standards* (AERA/APA/NCME, 2014) state that interpretation of subscores requires demonstration of the scores' "distinctiveness and reliability" (Standard 1.14), which do not appear to be present for the WJ IV Cognitive index scores. Thus, while emphasizing the interpretation of WJ IV Cognitive index scores may be well-intentioned, it may also be a practice that is psychometrically indefensible. Practitioners are cautioned to head the factor analytic evidence prior to engaging in the practice of regarding the interpretive utility of lower order factors (Dombrowski, 2015a). From a psychometric perspective, and despite the next wave of test interpretation books, guides, and chapters that will offer advice regarding this practice, there is a body of structural validity literature that suggests that the practice of interpreting lower order factors (i.e., index scores; CHC factors) must be undertaken only very cautiously due to structural validity concerns (see Dombrowski, 2013).

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Table 1

WJ-IV Cognitive Seven Factor Principal Axis Factor Extraction with Promax Rotation (Ages 9–13)

Subtest (Hypothesized CHC factor)	g loading	Pattern (Structure) Coefficients							h^2	u^2
		I	II	III	IV	V	VI	VII		
Verbal Attention (Gwm)	.63	.77 (.74)	-.20 (.35)	-.05 (.35)	.03 (.46)	.19 (.50)	-.05 (.21)	.05 (.17)	.60	.40
Memory for Words (Aud Mem)	.59	.75 (.69)	.11 (.45)	-.14 (.22)	-.13 (.33)	-.07 (.32)	.15 (.36)	.06 (.19)	.54	.44
Object Number Sequence (Gwm)	.69	.63 (.73)	.15 (.55)	.19 (.50)	-.04 (.41)	-.09 (.41)	-.03 (.19)	.02 (.17)	.57	.43
Nonword Repetition (Ga)	.52	.61 (.54)	.07 (.44)	.13 (.41)	.08 (.38)	-.11 (.42)	.03 (.22)	-.28 (.42)	.46	.54
Phonological Processing (Ga)	.64	.43 (.62)	-.09 (.38)	.11 (.38)	.10 (.47)	.02 (.44)	.30 (.48)	.08 (.27)	.50	.50
Picture Recognition (Gv)	.44	-.02 (.32)	.66 (.59)	.07 (.31)	.07 (.29)	-.15 (.19)	-.20 (-.34)	.05 (.06)	.41	.59
Visualization (Gv)	.58	.00 (.43)	.66 (.68)	.00 (.32)	.00 (.36)	.02 (.37)	.09 (.26)	-.03 (.05)	.48	.52
Visual-Auditory Learning (Glr)	.48	.02 (.36)	.56 (.55)	-.06 (.20)	-.02 (.28)	-.08 (.24)	.21 (.36)	.10 (.20)	.38	.62
Analysis-Synthesis (Gf)	.63	-.07 (.44)	.50 (.64)	.01 (.39)	-.01 (.39)	.26 (.53)	.17 (.38)	.07 (.20)	.51	.49
Story Recall (Glr)	.53	.14 (.46)	.37 (.54)	-.05 (.30)	.05 (.38)	.22 (.45)	-.08 (.11)	-.08 (-.01)	.36	.64
Pair Cancellation (Gs)	.49	-.01 (.32)	-.06 (.31)	.85 (.74)	.00 (.26)	-.10 (.35)	.13 (.13)	-.12 (.10)	.58	.42
Letter-Pattern Match (Gs)	.57	-.05 (.37)	.04 (.39)	.74 (.79)	.06 (.32)	-.02 (.42)	-.05 (.09)	.23 (.39)	.68	.32
Number-Pattern Match (PerSpd)	.56	.00 (.38)	.01 (.38)	.64 (.75)	-.13 (.24)	.30 (.57)	-.05 (.05)	-.01 (.16)	.62	.38
General Information (Gc)	.54	-.06 (.42)	.07 (.40)	-.06 (.24)	.83 (.78)	-.04 (.34)	-.02 (.22)	.05 (.09)	.63	.37
Oral Vocabulary (Gc)	.69	.04 (.56)	-.04 (.46)	.03 (.37)	.78 (.86)	.09 (.52)	.08 (.34)	-.03 (.08)	.77	.23
Number Series (Gf)	.62	.00 (.46)	-.02 (.40)	.06 (.47)	.02 (.42)	.69 (.76)	.08 (.32)	.08 (.22)	.61	.39
Concept Formation (Gf)	.63	.04 (.48)	.35 (.57)	.03 (.32)	.03 (.43)	.13 (.47)	.42 (.54)	-.11 (.08)	.53	.47
Numbers Reversed (Gwm)	.59	.30 (.54)	.13 (.44)	.04 (.41)	.05 (.38)	.10 (.42)	-.05 (.22)	.34 (.42)	.44	.56
Eigenvalue		6.57	1.64	1.27	1.08	0.95	0.80	0.76		
% Variance		36.51	9.15	7.10	6.03	5.29	4.48	4.27		
Factor Correlations	Factor 1	—								
	Factor 2	.61	—							
	Factor 3	.48	.47	—						
	Factor 4	.59	.51	.37	—					
	Factor 5	.56	.50	.55	.50	—				
	Factor 6	.31	.26	.10	.30	.29	—			
	Factor 7	.17	.10	.24	.09	.14	.28	—		

Note. h^2 = Communality coefficient. u^2 = Uniqueness. Pattern coefficients $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ-IV Technical Manual is indicated following each subtest name.

Table 2

WJ-IV Cognitive Seven Factor Principal Axis Factor Extraction with Promax Rotation (Ages 14–19)

Subtest (Hypothesized CHC factor)	g loading	Pattern (Structure) Coefficients							h^2	u^2
		I	II	III	IV	V	VI	VII ^a		
Verbal Attention (Gwm)	.65	.82 (.76)	-.02 (.43)	.11 (.51)	-.10 (.25)	.10 (.51)	-.22 (.35)	.02 (.19)	.63	.37
Memory for Words (Aud Mem)	.61	.80 (.72)	-.11 (.34)	-.13 (.36)	.11 (.41)	-.06 (.42)	.07 (.47)	.06 (.23)	.56	.44
Nonword Repetition (Ga)	.65	.66 (.60)	.00 (.29)	.02 (.39)	.07 (.35)	-.17 (.36)	.09 (.39)	-.26 (-.08)	.43	.57
Object Number Sequence (Gwm)	.73	.57 (.74)	.19 (.56)	-.06 (.45)	.15 (.49)	-.02 (.55)	.06 (.53)	-.02 (.19)	.60	.40
Numbers Reversed (Gwm)	.65	.33 (.62)	.04 (.49)	.03 (.44)	.07 (.37)	.20 (.53)	.01 (.48)	.29 (.41)	.50	.50
Pair Cancellation (Gs)	.54	-.04 (.40)	.81 (.74)	.00 (.35)	-.10 (.19)	-.10 (.38)	.16 (.39)	-.06 (.20)	.57	.43
Letter-Patten Match (Gs)	.62	-.05 (.46)	.73 (.78)	.03 (.37)	.16 (.38)	-.07 (.44)	.01 (.41)	.14 (.36)	.64	.36
Number-Pattern Match (PerSpd)	.55	.02 (.43)	.71 (.74)	-.06 (.30)	.01 (.29)	.27 (.54)	-.23 (.24)	-.03 (.14)	.61	.39
General Information (Gc)	.61	-.06 (.48)	-.05 (.34)	.85 (.80)	.13 (.30)	-.01 (.46)	-.04 (.47)	.05 (.12)	.66	.34
Oral Vocabulary (Gc)	.74	.04 (.62)	.04 (.47)	.76 (.89)	-.05 (.29)	.06 (.60)	.10 (.60)	-.08 (.07)	.81	.19
Picture Recognition (Gv)	.38	.04 (.29)	.04 (.22)	.04 (.17)	.66 (.61)	-.21 (.22)	.02 (.29)	.01 (.05)	.40	.60
Story Recall (Glr)	.58	.14 (.49)	-.01 (.34)	.11 (.41)	.38 (.58)	.23 (.55)	-.03 (.41)	-.13 (-.06)	.46	.54
Visualization (Gv)	.61	-.02 (.46)	-.04 (.36)	-.01 (.37)	.38 (.61)	.22 (.55)	.29 (.58)	.02 (.10)	.51	.49
Number Series (Gf)	.65	-.03 (.51)	.02 (.49)	.04 (.49)	-.15 (.29)	.82 (.77)	.02 (.47)	.11 (.17)	.63	.37
Analysis-Synthesis (Gf)	.64	-.05 (.48)	.02 (.44)	-.13 (.39)	.30 (.57)	.39 (.63)	.17 (.54)	.07 (.15)	.51	.49
Concept Formation (Gf)	.66	.00 (.51)	-.01 (.39)	-.07 (.46)	-.02 (.42)	.34 (.63)	.62 (.73)	-.10 (.05)	.60	.40
Visual-Auditory Learning (Glr)	.49	-.05 (.37)	.00 (.27)	.06 (.36)	.21 (.42)	-.12 (.33)	.58 (.61)	.01 (.15)	.41	.59
Phonological Processing (Ga)	.68	.32 (.64)	.08 (.49)	.14 (.59)	-.25 (.19)	.06 (.51)	.36 (.64)	.10 (.32)	.59	.41
Eigenvalue		7.19	1.44	1.26	1.11	0.85	0.84	0.70		
% Variance		36.96	8.03	7.02	6.19	4.76	4.67	3.92		
Factor Correlations	Factor 1	—								
	Factor 2	.58	—							
	Factor 3	.62	.46	—						
	Factor 4	.47	.36	.28	—					
	Factor 5	.65	.59	.58	.51	—				
	Factor 6	.62	.46	.59	.48	.59	—			
	Factor 7	.25	.30	.59	.03	.06	.24	—		

Note. h^2 = Communality coefficient. u^2 = Uniqueness. Pattern coefficients $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicted following each subtest name. ^a Factor VII is impermissible as it contains no salient subtest loadings ($b \geq .30$).

Table 3

Sources of WJ IV Subtest Variance According to Schmid-Leiman Orthogonalization of Seven Factors (Ages 9-13)

Subtest (Hypothesized CHC factor)	Second Order Factor		First Order Factors														h^2	u^2
	g		F1		F2		F3		F4		F5		F6		F7			
	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2		
Verbal Attention (Gwm)	.58	.33	.45	.21	-.14	.02	-.04	.00	.03	.00	.15	.02	-.05	.00	.06	.00	.59	.41
Mem for Words (Aud Mem)	.53	.28	.44	.19	.08	.01	-.10	.01	-.10	.01	-.06	.00	.14	.02	.06	.00	.53	.47
Object Num Sequence (Ga)	.67	.44	.37	.14	.10	.01	.13	.02	-.04	.00	-.08	.01	-.03	.00	.03	.00	.62	.39
Nonword Repetition (Ga)	.50	.25	.36	.13	.05	.00	.01	.00	.06	.00	-.09	.01	.04	.00	-.28	.08	.47	.53
Phonological Process (Ga)	.59	.35	.26	.07	-.06	.00	.08	.01	.08	.01	.02	.00	.28	.08	.08	.01	.51	.49
Picture Recognition (Gv)	.42	.17	-.01	.00	.45	.20	.05	.00	.05	.00	-.12	.01	-.19	.04	.05	.00	.44	.57
Visualization (Gv)	.52	.27	.00	.00	.45	.20	.00	.00	-.01	.00	.02	.00	.08	.01	-.04	.00	.48	.52
Vis Aud Learning (Glr)	.43	.18	.02	.00	.38	.14	-.05	.00	-.02	.00	-.07	.00	.20	.04	.11	.01	.38	.62
Analysis-Synthesis (Gf)	.56	.31	-.05	.00	.34	.11	.01	.00	-.01	.00	.21	.04	.16	.03	.07	.01	.50	.50
Story Recall (Glr)	.47	.22	.08	.01	.25	.06	-.04	.00	.04	.00	.17	.03	-.08	.01	-.08	.01	.34	.66
Pair Cancellation (Gs)	.53	.28	-.01	.00	-.04	.00	.59	.34	.00	.00	-.08	.01	.13	.02	-.12	.01	.66	.34
Letter-Pattern Match (Gs)	.59	.35	-.03	.00	.03	.00	.51	.26	.05	.00	-.02	.00	-.05	.00	.23	.05	.66	.34
Number-Pattern Match (PerSpd)	.55	.30	.00	.00	.01	.00	.44	.20	-.10	.01	.24	.06	-.05	.00	-.01	.00	.56	.44
General Information (Gc)	.51	.26	-.04	.00	.05	.00	-.04	.00	.61	.37	-.04	.00	-.02	.00	.05	.00	.64	.36
Oral Vocabulary (Gc)	.64	.41	.02	.00	-.03	.00	.02	.00	.57	.33	.07	.01	.08	.01	-.04	.00	.75	.25
Number Series (Gf)	.53	.28	.00	.00	-.02	.00	.04	.00	.02	.00	.54	.29	.08	.01	.08	.01	.59	.41
Concept Formation (Gf)	.56	.31	.03	.00	.24	.06	.03	.00	.02	.00	.10	.01	.39	.15	-.11	.01	.54	.46
Numbers Reversed (Gwm)	.54	.29	.18	.03	.09	.01	.03	.00	.04	.00	.08	.01	-.05	.00	.33	.11	.45	.55
Common Variance		.55		.08		.09		.09		.08		.05		.04		.03	.54	.46
Total Variance		.27		.04		.04		.04		.04		.03		.02		.02		
ω_H/ω_{HS}		.83		.27		.28		.37		.42		.29		.16		.11		

Note. b = factor loading, S^2 = variance explained, h^2 = communality, u^2 = uniqueness, ω_H = Omega hierarchical (g), ω_{HS} = Omega-hierarchical subscale (group factors F1-F7). Loadings $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

Table 4

Sources of WJ IV Subtest Variance According to Schmid-Leiman Orthogonalization of Seven Factors (Ages 14-19)

Subtest (Hypothesized CHC factor)	Second Order Factor		First Order Factors														h^2	u^2
	g		F1		F2		F3		F4		F5		F6		F7			
	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2	b	S^2		
Verbal Attention (Gwm)	.64	.41	.44	.19	-.01	.00	.07	.00	-.08	.01	.08	.01	-.19	.04	.03	.00	.66	.34
Mem Words (Aud Mem Span)	.57	.32	.43	.18	-.07	.00	-.09	.01	.08	.01	-.05	.00	.06	.00	.06	.00	.54	.46
Nonword repetition (Ga)	.51	.26	.36	.13	.00	.00	.02	.00	.05	.00	-.12	.02	.08	.01	-.26	.07	.48	.52
Object Num Sequence (Gwm)	.72	.51	.31	.10	.11	.01	-.04	.00	.11	.01	-.01	.00	.05	.00	-.02	.00	.64	.36
Num Reversed (Gwm)	.61	.37	.18	.03	.03	.00	.02	.00	.05	.00	.15	.02	.01	.00	.28	.08	.51	.49
Pair Cancellation (Gs)	.55	.30	-.03	.00	.48	.23	.00	.00	-.07	.01	-.08	.01	.14	.02	-.06	.00	.57	.44
Letter-Pattern Match (Gs)	.67	.45	-.03	.00	.43	.18	.02	.00	.11	.01	-.05	.00	.02	.00	.14	.02	.68	.32
Num-Pat Match (Perc Speed)	.61	.37	.01	.00	.42	.18	-.04	.00	.01	.00	.20	.04	-.19	.04	-.04	.00	.63	.37
General Information (Gc)	.64	.41	-.03	.00	-.03	.00	.55	.30	.10	.01	-.01	.00	-.04	.00	.06	.00	.72	.28
Oral Vocabulary (Gc)	.72	.51	.03	.00	.03	.00	.49	.24	-.04	.00	.05	.00	.08	.01	-.08	.01	.77	.23
Picture Recognition (Gv)	.45	.20	.03	.00	.02	.00	.03	.00	.47	.22	-.16	.02	.02	.00	.01	.00	.45	.55
Story Recall (Glr)	.58	.34	.08	.01	-.01	.00	.07	.01	.27	.08	.17	.03	-.03	.00	-.13	.02	.47	.53
Visualization (Gv)	.54	.29	-.01	.00	-.03	.00	-.01	.00	.27	.08	.17	.03	.25	.06	.03	.00	.45	.55
Number Series (Gf)	.52	.27	-.02	.00	.01	.00	.03	.00	-.11	.01	.60	.35	.02	.00	.11	.01	.65	.35
Analysis-Synthesis (Gf)	.57	.32	-.03	.00	.02	.00	-.01	.00	.21	.05	.29	.08	.14	.02	.08	.01	.48	.52
Concept Formation (Gf)	.46	.21	.00	.00	-.01	.00	-.05	.00	-.02	.00	.25	.06	.52	.27	-.10	.01	.55	.45
Visual-Auditory Lrn (Glr)	.39	.15	-.03	.00	.00	.00	.04	.00	.15	.02	-.09	.01	.48	.23	.02	.00	.42	.58
Phonological Proc (Ga)	.56	.31	.18	.03	.05	.00	.03	.00	-.18	.03	.05	.00	-.19	.04	.10	.01	.49	.51
Common Variance		.59		.07		.06		.06		.05		.07		.08		.02	.56	.44
Total Variance		.30		.03		.03		.03		.03		.03		.04		.01		
ω_H/ω_{HS}		.85		.23		.28		.31		.19		.27		.27		.08		

Note. b = factor loading, S^2 = variance explained, h^2 = communality, u^2 = uniqueness, ω_H = Omega hierarchical (g), ω_{HS} = Omega-hierarchical subscale (group factors F1-F7). Loadings $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

Table 5

WJ IV Cognitive Principal Axis Factor Extraction (Four Factors) with Promax Rotation (Ages 9-13)

Subtest (Hypothesized CHC factor)	g loading	Factor Pattern (Structure) Coefficients				h^2	u^2
		F1	F2	F3	F4		
Verbal Attention (Gwm)	.63	.84 (.74)	-.24 (.39)	.04 (.41)	.06 (.48)	.58	.42
Memory for Words (Aud Mem)	.59	.82 (.70)	.13 (.49)	-.18 (.26)	-.16 (.35)	.54	.44
Object Number Sequence (Gwm)	.69	.61 (.71)	.11 (.55)	.17 (.51)	-.10 (.42)	.54	.44
Nonword Repetition (Ga)	.52	.55 (.55)	.07 (.47)	.13 (.46)	.07 (.40)	.35	.65
Phonological Processing (Ga)	.63	.52 (.65)	-.02 (.46)	.09 (.43)	.13 (.50)	.44	.56
Numbers Reversed (Gwm)	.59	.35 (.55)	.10 (.47)	.20 (.46)	.02 (.40)	.35	.65
Visualization (Gv)	.58	-.02 (.44)	.73 (.69)	.02 (.34)	-.01 (.38)	.48	.52
Visual-Auditory Learning (Glr)	.48	.05 (.39)	.62 (.58)	-.10 (.24)	-.04 (.30)	.35	.65
Picture Recognition (Gv)	.43	-.11 (.30)	.58 (.53)	.06 (.30)	-.02 (.27)	.29	.71
Analysis-Synthesis (Gf)	.63	.00 (.49)	.58 (.67)	.12 (.45)	.05 (.44)	.47	.53
Concept Formation (Gf)	.62	.14 (.53)	.47 (.63)	.01 (.37)	.12 (.48)	.43	.57
Story Recall (Glr)	.53	.13 (.45)	.37 (.53)	.02 (.34)	.09 (.40)	.30	.70
Letter-Pattern Match (Gs)	.57	-.07 (.38)	.01 (.40)	.82 (.79)	.00 (.33)	.63	.37
Number-Pattern Match (PerSpd)	.56	.00 (.39)	.01 (.39)	.80 (.77)	-.10 (.28)	.60	.40
Pair Cancellation (Gs)	.49	-.04 (.33)	-.03 (.32)	.75 (.69)	-.01 (.27)	.29	.71
Number Series (Gf)	.61	.15 (.51)	.06 (.46)	.35 (.55)	.18 (.48)	.39	.61
Oral Vocabulary (Gc)	.69	.06 (.59)	-.03 (.50)	.01 (.42)	.85 (.87)	.77	.23
General Information (Gc)	.54	-.07 (.43)	.04 (.42)	-.09 (.27)	.82 (.76)	.59	.41
Eigenvalue		6.57	1.64	1.27	1.08		
% Variance		36.51	9.15	7.10	6.03		
Factor Correlations	Factor 1	—					
	Factor 2	.67	—				
	Factor 3	.55	.53	—			
	Factor 4	.63	.57	.46	—		

Note. h^2 = Communality coefficient. u^2 = Uniqueness. Pattern coefficients $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

Table 6

Sources of WJ IV Subtest Variance According to a Schmid-Leiman Orthogonalization of Four Factors (Ages 9-13)

Subtest (Hypothesized CHC factor)	Second-Order Factor		First Order Factors									
	<i>g</i>		F1		F2		F3		F4		<i>h</i> ²	<i>u</i> ²
	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²		
Verbal Attention (Gwm)	.60	.36	.44	.19	-.15	.02	.03	.00	.05	.00	.58	.42
Memory for Words (Aud Mem)	.57	.33	.42	.18	.08	.01	-.13	.02	-.12	.01	.54	.46
Object Number Sequence (Gwm)	.67	.45	.32	.10	.07	.00	.12	.01	-.08	.01	.57	.43
Nonword Repetition (Ga)	.49	.24	.29	.08	.05	.00	-.09	.01	.06	.00	.33	.67
Phonological Processing (Ga)	.59	.35	.27	.07	-.01	.00	.06	.00	.11	.01	.44	.56
Numbers Reversed (Gwm)	.55	.31	.18	.03	.07	.00	.14	.02	.02	.00	.36	.64
Visualization (Gv)	.54	.29	-.01	.00	.44	.19	-.02	.00	-.01	.00	.48	.52
Visual-Auditory Learning (Glr)	.45	.20	.03	.00	.38	.14	-.07	.00	-.04	.00	.35	.65
Picture Recognition (Gv)	.40	.16	-.06	.00	.35	.13	.05	.00	-.02	.00	.30	.71
Analysis-Synthesis (Gf)	.59	.35	.00	.00	.35	.12	.08	.01	.04	.00	.48	.52
Concept Formation (Gf)	.57	.33	.07	.01	.28	.08	-.01	.00	.10	.01	.42	.58
Story Recall (Glr)	.49	.24	.07	.01	.22	.05	.02	.00	.07	.00	.30	.70
Letter-Pattern Matching (Gs)	.55	.30	-.04	.00	.01	.00	.57	.32	.00	.00	.62	.38
Number-Pattern Matching (PerSpd)	.54	.29	.00	.00	.01	.00	.55	.31	-.08	.01	.60	.40
Pair Cancellation (Gs)	.47	.22	-.02	.00	-.02	.00	.52	.27	-.01	.00	.49	.52
Number Series (Gf)	.56	.32	.08	.01	.04	.00	.24	.06	.14	.02	.40	.60
Oral Vocabulary (Gc)	.59	.35	.03	.00	-.02	.00	.01	.00	.65	.42	.77	.23
General Information (Gc)	.45	.20	-.04	.00	.03	.00	-.06	.00	.63	.39	.60	.40
Common Variance		.610		.079		.088		.120		.104	.479	.521
Total Variance		.238		.043		.033		.072		.054		
ω _H /ω _{HS}		.797		.213		.239		.355		.489		

Note. b = factor loading, S^2 = variance explained, h^2 = communality, u^2 = uniqueness, ω_H = Omega hierarchical (g), ω_{HS} = Omega-hierarchical subscale (group factors F1-F4). Loadings $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

Table 7

WJ IV Cognitive Principal Axis Factor Extraction (Four Factors) with Promax Rotation (Ages 14-19)

Subtest (Hypothesized CHC factor)	g loading	Factor Pattern (Structure) Coefficients				h^2	u^2
		F1	F2	F3	F4		
Visualization (Gv)	.61	.71 (.71)	-.01 (.41)	-.02 (.45)	.04 (.46)	.51	.49
Picture Recognition (Gv)	.37	.64 (.51)	-.04 (.22)	.08 (.30)	-.25 (.17)	.29	.71
Analysis-Synthesis (Gf)	.64	.57 (.68)	.14 (.50)	.03 (.47)	.07 (.50)	.48	.52
Visual-Auditory Learning (Glr)	.49	.51 (.55)	-.07 (.30)	.05 (.35)	.19 (.43)	.32	.68
Concept Formation (Gf)	.66	.49 (.66)	.01 (.45)	.05 (.48)	.31 (.59)	.49	.51
Story Recall (Glr)	.58	.48 (.60)	-.02 (.38)	.14 (.48)	.06 (.45)	.38	.62
Number-Pattern Match (PerSpd)	.55	-.05 (.37)	.83 (.74)	.01 (.42)	-.09 (.37)	.56	.44
Letter-Pattern Match (Gs)	.62	.09 (.47)	.78 (.76)	-.03 (.45)	-.07 (.42)	.59	.41
Pair Cancellation (Gs)	.53	-.08 (.34)	.78 (.71)	-.08 (.38)	.05 (.41)	.51	.49
Verbal Attention (Gwm)	.65	-.24 (.38)	.03 (.48)	.79 (.76)	.16 (.55)	.61	.39
Memory for Words (Aud Mem)	.62	.16 (.51)	-.11 (.39)	.76 (.72)	-.11 (.42)	.55	.45
Nonword Repetition (Ga)	.65	.11 (.43)	-.14 (.31)	.56 (.59)	.05 (.42)	.36	.64
Object Number Sequence (Gwm)	.73	.19 (.61)	.18 (.60)	.54 (.74)	-.07 (.52)	.60	.40
Numbers Reversed (Gwm)	.65	.11 (.51)	.21 (.54)	.34 (.61)	.09 (.52)	.43	.57
Oral Vocabulary (Gc)	.74	-.06 (.52)	-.02 (.50)	.03 (.58)	.92 (.88)	.79	.21
General Information (Gc)	.60	.00 (.44)	-.10 (.37)	.00 (.46)	.80 (.73)	.55	.45
Phonological Processing (Ga)	.68	-.05 (.47)	.13 (.53)	.27 (.61)	.44 (.67)	.51	.49
Number Series (Gf)	.64	.12 (.51)	.28 (.57)	-.01 (.48)	.36 (.60)	.44	.56
Eigenvalue		7.19	1.44	1.26	1.11		
% Variance		39.96	8.03	7.02	6.19		
Factor Correlations	Factor 1	—					
	Factor 2	.58	—				
	Factor 3	.64	.62	—			
	Factor 4	.62	.59	.66	—		

Note. h^2 = Communality coefficient. u^2 = Uniqueness. Pattern coefficients $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

Table 8

Sources of WJ IV Subtest Variance According to a Schmid-Leiman Orthogonalization of Four Factors (Ages 14-19)

Subtest (Hypothesized CHC factor)	Second-Order Factor		First Order Factors									
	<i>g</i>		F1		F2		F3		F4		<i>h</i> ²	<i>u</i> ²
	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²	<i>b</i>	<i>S</i> ²		
Visualization (Gv)	.60	.36	.40	.16	-.01	.00	-.02	.00	.03	.00	.52	.49
Picture Recognition (Gv)	.57	.33	.36	.13	-.03	.00	.05	.00	-.17	.03	.30	.70
Analysis-Synthesis (Gf)	.67	.45	.33	.11	.09	.01	-.02	.00	.05	.00	.50	.50
Visual-Auditory Learning (Glr)	.49	.24	.29	.08	-.05	.00	-.04	.00	.13	.02	.32	.68
Concept Formation (Gf)	.59	.35	.28	.08	.01	.00	-.03	.00	.21	.04	.50	.50
Story Recall (Glr)	.55	.31	.28	.08	-.02	.00	.09	.01	.05	.00	.38	.62
Number-Pattern Match (PerSpd)	.54	.29	-.03	.00	.50	.25	.01	.00	-.06	.00	.56	.44
Letter-Pattern Match (Gs)	.45	.20	.05	.00	.47	.22	-.02	.00	-.05	.00	.61	.39
Pair Cancellation (Gs)	.40	.16	-.05	.00	.47	.22	-.06	.00	.04	.00	.51	.49
Verbal Attention (Gwm)	.59	.35	-.14	.02	.02	.00	.50	.25	.11	.01	.60	.40
Memory for Words (Aud Mem)	.57	.33	.09	.01	-.07	.00	.48	.23	-.08	.01	.56	.44
Nonword Repetition (Ga)	.49	.24	.07	.00	-.09	.01	.36	.13	.04	.00	.36	.64
Object Number Sequence (Gwm)	.55	.30	.11	.01	.11	.01	.34	.12	-.05	.00	.62	.38
Numbers Reversed (Gwm)	.54	.29	.06	.00	.13	.02	.22	.05	.06	.00	.44	.56
Oral Vocabulary (Gc)	.47	.22	-.04	.00	-.02	.00	.02	.00	.61	.38	.79	.21
General Information (Gc)	.56	.32	.01	.00	-.06	.00	-.01	.00	.53	.28	.55	.45
Phonological Processing (Ga)	.59	.35	-.03	.00	.08	.01	.17	.03	.30	.09	.49	.51
Number Series (Gf)	.45	.20	.07	.01	.17	.03	-.01	.00	.24	.06	.45	.55
Common Variance		.643		.077		.086		.090		.103	.503	.497
Total Variance		.291		.035		.039		.041		.047		
ω _H /ω _{HS}		.805		.203		.336		.261		.306		

Note. b = factor loading, S^2 = variance explained, h^2 = communality, u^2 = uniqueness, ω_H = Omega hierarchical (g), ω_{HS} = Omega-hierarchical subscale (group factors F1-F4). Loadings $\geq .30$ are **bolded** (Carroll, 1993, p. 108; Child, 2006). Note that alignment of subtests with respective CHC stratum I or II factors posited in the WJ IV Technical Manual is indicated following each subtest name.

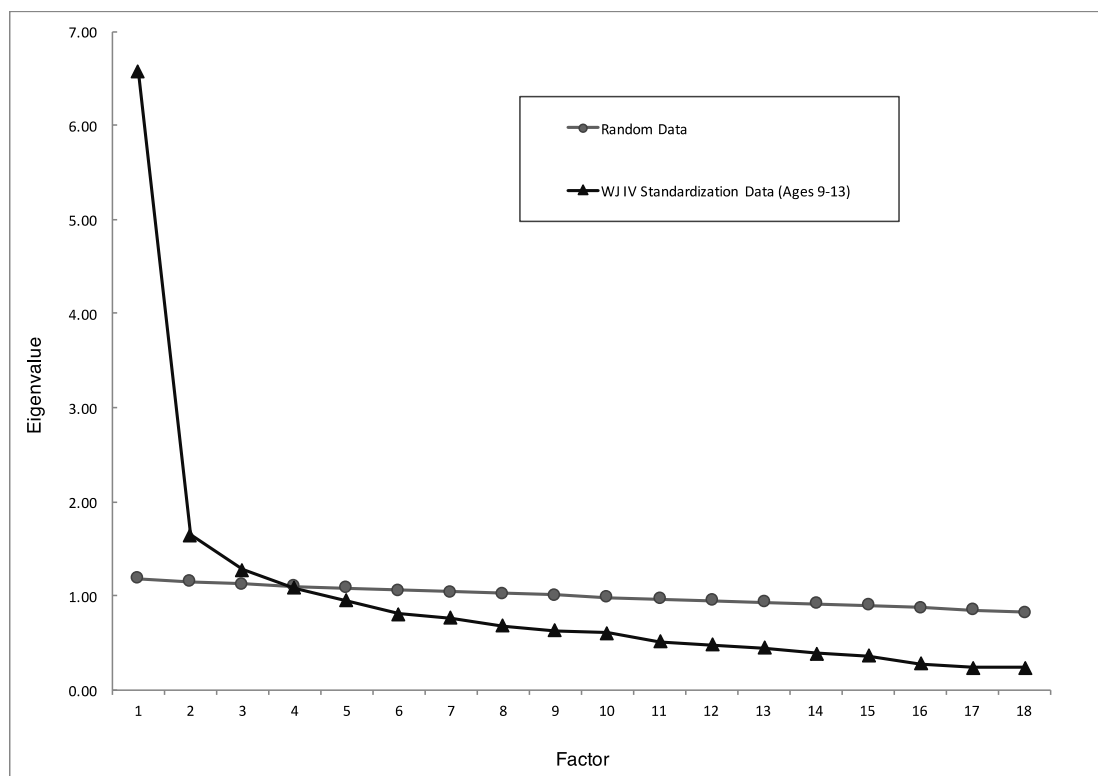


Figure 1. Scree plot of parallel analysis for WJ IV subtests (Ages 9-13).

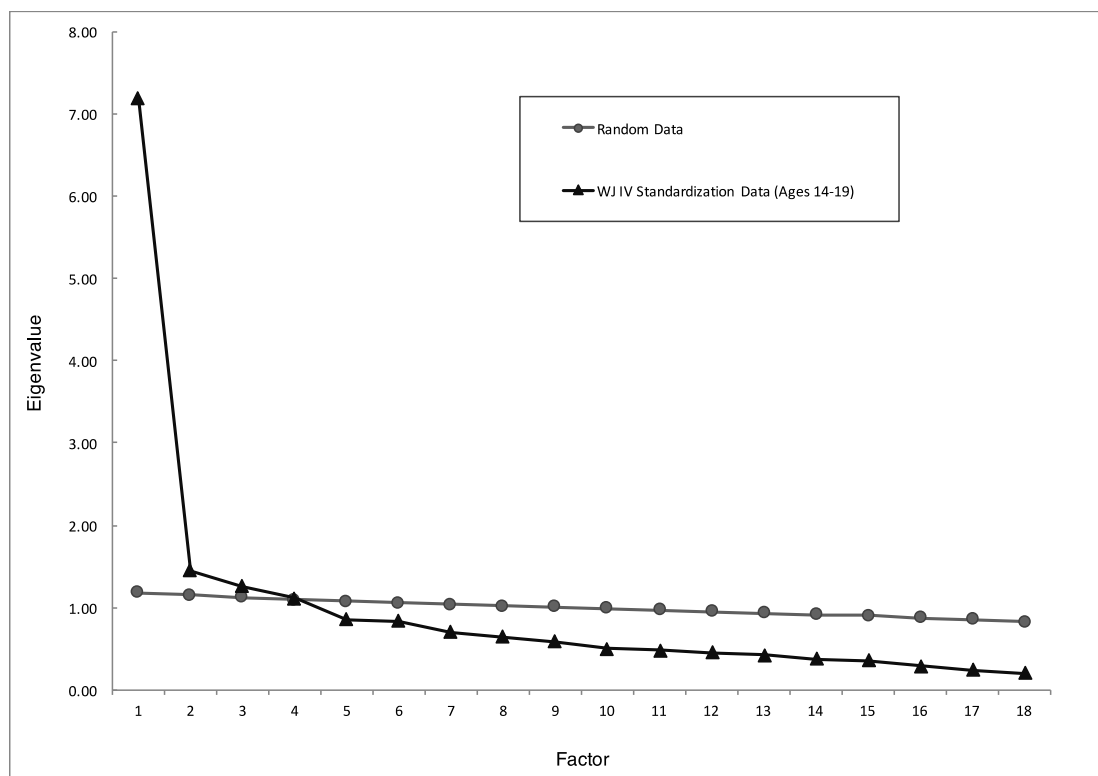


Figure 2. Scree plot of parallel analysis for WJ IV subtests (Ages 14-19).